

ADA103144

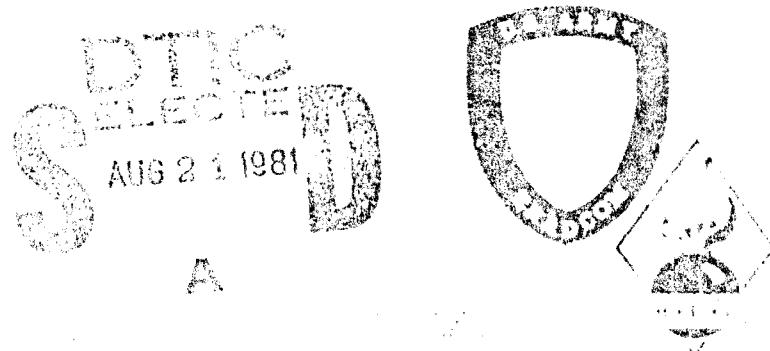
REF ID: A642

June 1981

(12)

A FORTRAN 77 Computer Program for Designing and
Implementing Recursive Digital Filters

by Vincent F. Gray
David R. Giambri



U.S. Army Electronics Research
and Development Command
Harry Diamond Laboratories

Adelphi, MD 20783

Approved for public release; distribution unlimited

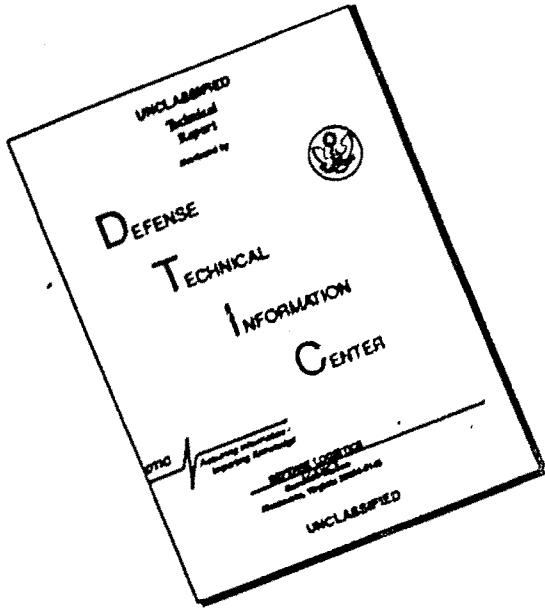
81 8 20 1981

The findings in this report are not to be construed as an official Department of the Army position unless so designated by other authorized documents.

Citation of manufacturers' or trade names does not constitute an official indorsement or approval of the use thereof.

Destroy this report when it is no longer needed. Do not return it to the originator.

DISCLAIMER NOTICE



**THIS DOCUMENT IS BEST
QUALITY AVAILABLE. THE COPY
FURNISHED TO DTIC CONTAINED
A SIGNIFICANT NUMBER OF
PAGES WHICH DO NOT
REPRODUCE LEGIBLY.**

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER HDL-TR-1942	2. GOVT ACCESSION NO. AD-A103144	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A FORTRAN IV Computer Program for Designing and Implementing Recursive Digital Filters.		5. TYPE OF REPORT & PERIOD COVERED Technical Report
7. AUTHOR(s) Robert F. Gray David R. Gambrel (IBM Corp.)		6. PERFORMING ORG. REPORT NUMBER
9. PERFORMING ORGANIZATION NAME AND ADDRESS Harry Diamond Laboratories 2800 Powder Mill Road Adelphi, MD 20783		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Program Ele: 6.21.20.A
11. CONTROLLING OFFICE NAME AND ADDRESS U.S. Army Materiel Development and Readiness Command Alexandria, VA 22333		12. REPORT DATE June 1981
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		13. NUMBER OF PAGES 70
16. DISTRIBUTION STATEMENT (of this Report)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
Approved for public release. Distribution unlimited.		
18. SUPPLEMENTARY NOTES DRCMS Code: 612120.H.250011 HDL Proj: X750E3		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Recursive digital filter Butterworth digital filter Filter design Chebyshev digital filter Bilinear transformation Elliptic digital filter		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A FORTRAN IV computer program has been developed that will design and implement recursive digital filters. Standard low pass analog designs are used in combination with the bilinear transform to design the filters. The analog designs used are Butterworth (maximally flat), Chebyshev (equiripple in the passband), and elliptic (equiripple in both passband and stop band). Transformations are possible in the z domain to obtain high-pass,		

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

20. Abstract (Cont'd)

band-pass, and band-stop filters from the low-pass designs. The development of the computer program is detailed along with all the necessary equations. A complete listing of the code also is provided.

UNCLASSIFIED

2 SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

FOREWORD

This report resulted from a joint project assigned to the authors for a course on digital signal processing offered at the George Washington University, Washington, DC. The authors thank Professor N. Kyriakopoulos for his guidance during the project.

CONTENTS

	<u>Page</u>
FOREWORD	3
1. INTRODUCTION	7
2. APPROACH	7
2.1 Invariant-Impulse-Response Method	8
2.2 Matched z Transformation	8
2.3 Bilinear Transformation	9
2.4 Analog Filter Designs	11
2.4.1 Butterworth	11
2.4.2 Chebyshev	12
2.4.3 Elliptic	12
2.5 Digital Filter Realization	13
2.6 Band Transformations	13
3. FORTRAN IV PROGRAM	14
3.1 Outline	14
3.1.1 RDATA	14
3.1.2 WARP	16
3.1.3 FACTOR	16
3.1.4 FPLOT	17
3.1.5 Filter Choice	17
3.1.6 FILIMP	17
3.1.7 PDATA	18
3.2 Program Use	18
4. DESIGN EXAMPLES	24
4.1 Analog Design	24
4.2 Band Transformations	24
4.3 Data Processing	29
5. CONCLUSIONS AND RECOMMENDATIONS	32
LITERATURE CITED	33
SELECTED BIBLIOGRAPHY	34
DISTRIBUTION	65

CONTENTS (Cont'd)

	<u>Page</u>
APPENDICES	
A---EQUATIONS USED FOR CODE DEVELOPMENT	35
B---FORTRAN IV COMPUTER PROGRAM FOR RECURSIVE DIGITAL FILTERS	49

FIGURES

1 Bilinear transformation	10
2 Recursive digital filter in parallel form	13
3 Recursive digital filter program	15
4 Input data for example 1	21
5 Output data for example 1	21
6 Low-pass Cauer filter: amplitude versus frequency	22
7 Low-pass Cauer filter: phase versus frequency	22
8 Band-pass Cauer filter: amplitude versus frequency	23
9 Band-pass Cauer filter: phase versus frequency	23
10 Low-pass Butterworth filter: amplitude versus frequency	25
11 Low-pass Butterworth filter: phase versus frequency	25
12 Low-pass Chebyshev filter: amplitude versus frequency	26
13 Low-pass Chebyshev filter: phase versus frequency	26
14 High-pass Butterworth filter: amplitude versus frequency	27
15 High-pass Butterworth filter: phase versus frequency	27
16 Band-stop Chebyshev filter: amplitude versus frequency	28
17 Band-stop Chebyshev filter: phase versus frequency	28
18 Input waveform obtained by summing three sinusoids	29
19 Output waveform obtained by using low-pass Butterworth filter	30
20 Output waveform obtained by using high-pass Butterworth filter	30
21 Output waveform obtained by using band-pass Cauer filter	31
22 Output waveform obtained by using band-stop Chebyshev filter	31

1. INTRODUCTION

The purpose of this project is to develop a computer program that will design and implement a recursive digital filter to meet user specifications given as inputs to the program. The filter type is selected from three well-known analog filter approximations: (1) Butterworth, maximally flat; (2) Chebyshev, equiripple in the passband; and (3) elliptic (or Cauer), equiripple in both the passband and the stop band. For a low-pass filter, for example, the user specifies the filter type, the filter order, the cutoff frequency, and the sampling rate. High-pass, band-pass, and band-stop filters also may be chosen for design.

The program calculates the poles and the zeros of the low-pass filter in the analog s domain. The bilinear transformation technique is then used to determine the coefficients of the transfer function in the z domain. If a high-pass, band-pass, or band-stop filter is required, then the coefficients are determined by a transformation in the z domain of the prototype low-pass digital filter whose coefficients have been determined from an analog low-pass filter using the bilinear transformation.

After determining the coefficients of the digital filter transfer function, a plot of the magnitude and the phase may be obtained if desired by the user. A list of the coefficients also is printed. Another option is to implement the filter as a system of linear difference equations and to process input data consisting of sample values at any given sample rate through the filter. The filtered output data points are then printed as output.

In summary, the program performs the following:

- a. Calculates and prints the coefficients of the desired digital filter.
- b. Plots the magnitude and phase responses.
- c. Implements the digital filter and processes data through the filter.
- d. Outputs the filtered data.

2. APPROACH

There are two general types of digital filters, recursive and non-recursive. The output response of a nonrecursive filter is a function

of only the present and past values of the input excitation. A recursive filter is one in which the present output response is a function of the present and past values of the input, as well as past values of the output.

As in analog filters, the approximation step in the design of digital filters is the process whereby a realizable transfer function satisfying prescribed specifications is obtained. To be realizable as a recursive filter, a transfer function must satisfy the following constraints.

- a. It must be a rational function of z with real coefficients.
- b. Its poles must lie within the unit circle of the z plane.
- c. The degree of the numerator polynomial must be equal to or less than that of the denominator polynomial.

Recursive filter approximations can be obtained from analog filter approximations by using the following methods, all of which satisfy the above constraints:

Invariant-impulse-response method
Matched z transformation
Bilinear transformation

Each method has certain advantages and disadvantages.

2.1 Invariant-Impulse-Response Method

Given an analog filter transfer function, $H_A(s)$, the invariant-impulse-response method is implemented as follows:

- a. Obtain the impulse response of analog filter $H_A(t)$.
- b. Replace t by nT in $H_A(t)$.
- c. Form the z transform of $H_A(nT)$. This gives $H_D(z)$, the digital filter transfer function. T is the sample period = $1/f_s$.

This method gives good results if $H_A(j\omega) \approx 0$ for $\omega > \omega_s/2$. However, aliasing errors tend to restrict this method to the design of all pole filters.

2.2 Matched z Transformation

Given continuous-time transfer function

$$H_A(s) = \frac{H_0 \prod_{i=1}^M (s - s_i)}{\prod_{i=1}^N (s - p_i)},$$

a corresponding digital transfer function can be formed as

$$H_D(z) = (z + 1)^L \frac{H_0 \prod_{i=1}^M (z - e^{s_i T})}{\prod_{i=1}^N (z - e^{p_i T})},$$

where L is an integer equal to the number of zeros at $s = \infty$ in $H_A(s)$. This method gives reasonable results for high-pass and band-stop filters, although it tends to distort the passband ripple in Chebyshev and elliptic filters. For low-pass and band-pass filters, better approximations can be obtained by using the modified invariant-impulse-response method.¹

2.3 Bilinear Transformation

The bilinear transformation method yields a digital filter with approximately the same time-domain response as the original analog filter for any excitation.

$$H_D(z) = H_A(s) \Bigg|_{s = \frac{2}{T} \frac{z - 1}{z + 1}}$$

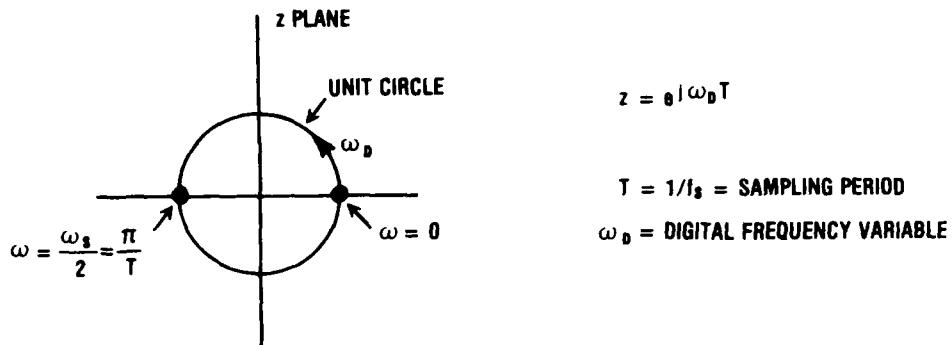
The bilinear transformation maps these:

- a. The open right-half s plane onto the region exterior to the unit circle $|z| = 1$ of the z plane
- b. The j axis of the s plane onto the unit circle $|z| = 1$
- c. The open left-half s plane onto the interior of the unit circle

From property c it follows that a stable analog filter yields a stable digital filter and, since the transformation has real coefficients, $H_D(z)$ has real coefficients.

¹A. Antoniou, *Digital Filters: Analysis and Design*, McGraw-Hill Book Co., Inc., New York (1979).

Digital filters obtained by the bilinear transformation do not suffer from the effects of aliasing. However, a nonlinear frequency distortion is introduced because the transformation maps the entire $j\omega$ axis onto the unit circle (fig. 1).



THE ANALOG FREQUENCY VARIABLE, ω_A , IS RELATED TO THE DIGITAL FREQUENCY VARIABLE, ω_b , THROUGH THE BILINEAR TRANSFORMATION:

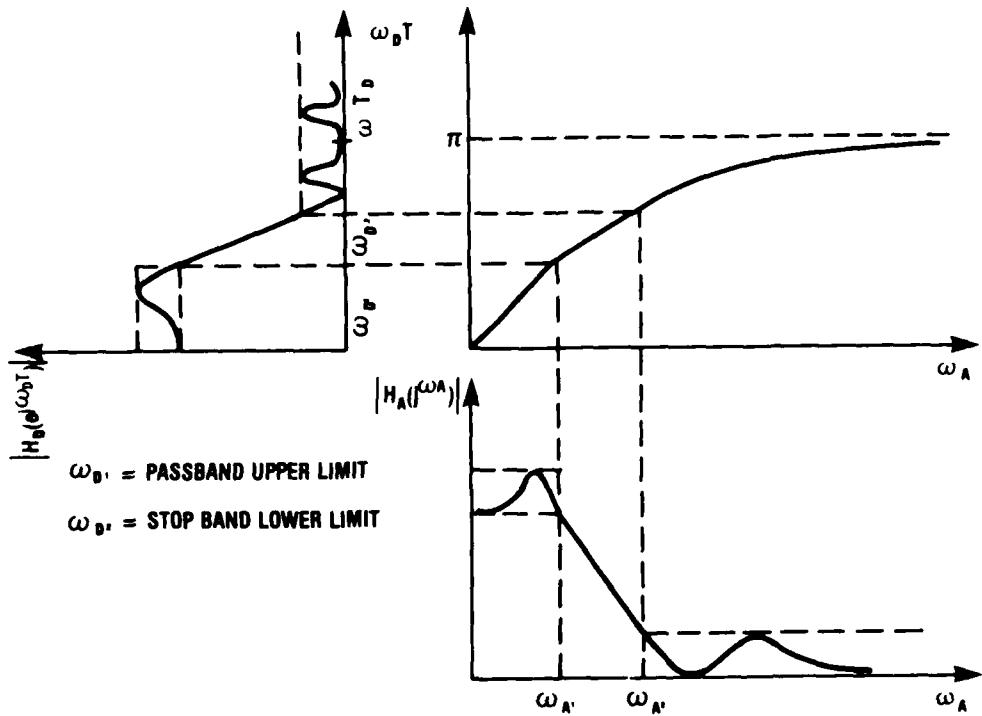


Figure 1. Bilinear transformation.

If only the amplitude response is of concern, the warping effect can for all practical purposes be eliminated by prewarping the analog filter. For example, if a low-pass cutoff frequency of ω_{DC} is desired for the digital filter, then the analog filter is first designed for an unnormalized cutoff frequency given by

$$\omega_{AC} = \frac{2}{T} \tan\left(\frac{\omega_{DC}^T}{2}\right).$$

The phase response of the derived digital filter is nonlinear because of the warping effect. Furthermore, little can be done to linearize it except by employing delay equalization. Consequently, if it is mandatory to preserve a linear phase response, alternative methods should be considered.

The bilinear transformation is the most important of the techniques used to obtain digital recursive filters from analog filters. The passband and loss characteristics of the analog filter are preserved, and there is no aliasing effect. Frequency distortion is compensated at ω_{DC} , and the transfer function can be obtained by a relatively easy transformation. For these reasons, the bilinear transformation was chosen as the method of obtaining the digital filter transfer function for this project.

2.4 Analog Filter Designs

The theory of analog filter approximations has been extensively developed.¹⁻⁴

2.4.1 Butterworth

The Butterworth approximation is the simplest all-pole type. In the stop band, the attenuation characteristics monotonically decrease as a function of frequency. The Butterworth filter is termed also a maximally flat magnitude approximation in that the error in the passband is also a monotonically decreasing function. The equations for calculating the pole locations are given in appendix A.

¹A. Antoniou, *Digital Filters: Analysis and Design*, McGraw-Hill Book Co., Inc., New York (1979).

²M. S. Ghausi, *Principles and Design of Linear Active Circuits*, McGraw-Hill Book Co., Inc., New York (1965), ch. 4.

³H. Y.-F. Lam, *Analog and Digital Filters*, Prentice-Hall, Inc., Englewood Cliffs, NJ (1979).

⁴D. E. Johnson, *Introduction to Filter Theory*, Prentice-Hall, Inc., Englewood Cliffs, NJ (1976).

2.4.2 Chebyshev

A second approximation that improves on the rate of change of the attenuation between passband and stop band over that of the Butterworth filter is the Chebyshev filter. The error in the passband is distributed evenly in an oscillating manner. This is called an equiripple approximation. In the stop band, the magnitude decreases monotonically with a faster cutoff rate than that of a Butterworth filter of the same order. The Chebyshev filter is an all-pole type and is based on the nth-order Chebyshev polynomials, $C_n(\omega)$.

$$C_n(\omega) = \begin{cases} \cos(n \cos^{-1} \omega), & 0 < \omega < 1, \\ \cosh(n \cosh^{-1} \omega), & \omega > 1. \end{cases}$$

The recursion formula for finding the nth-order polynomial is

$$C_0(\omega) = 1,$$

$$C_1(\omega) = \omega,$$

$$C_n(\omega) = 2\omega C_{n-1}(\omega) - C_{n-2}(\omega).$$

The equations for calculating the pole locations are given in appendix A.

2.4.3 Elliptic

The third type of filter approximation considered in this report is called the elliptic approximation and was first introduced by Wilhelm Cauer. This approximation is based on the Jacobi elliptic sine functions. In this approximation, the error in the passband is again distributed evenly in an oscillating manner. However, instead of a monotonically decreasing characteristic in the stop band, the stop-band attenuation oscillates between infinity and a prescribed maximum. Thus, there is an equiripple characteristic in both passband and stop band. The elliptic approximation is more efficient than the Butterworth and Chebyshev approximations in that the transition between passband and stop band is steeper for a given filter order. An elliptic filter transfer function has both poles and zeros and has been shown to be optimal in the sense of having the sharpest transition of any approximation. The technique used to calculate the coefficients for the elliptic filter is given in appendix A and was taken from the development by Antoniou.¹

¹A. Antoniou, *Digital Filters: Analysis and Design*, McGraw-Hill Book Co., Inc., New York (1979).

2.5 Digital Filter Realization

There are two possible canonical forms of linear difference equations that can be realized from the z domain transfer function, $H(z)$. One, the cascade form, follows from the factored form of $H(z)$. The other, the parallel form, requires the expansion of $H(z)$ into partial fractions. The parallel form was chosen for this project because of its reduced sensitivity to noise.

Realization of a recursive digital filter in parallel form is shown in figure 2.

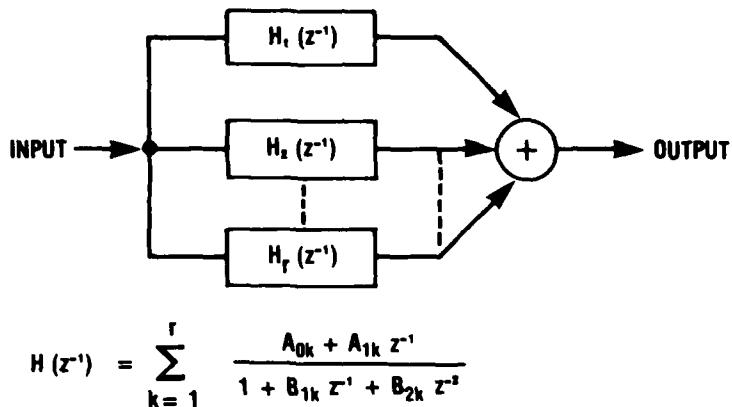


Figure 2. Recursive digital filter in parallel form.

2.6 Band Transformations

For this project, high-pass, band-pass, and band-stop filters are obtained from the digital low-pass prototype by a transformation in the z domain. Another approach is to design the analog low-pass prototype, perform the transformation in the s domain to high pass, band pass, or band stop, and then transform the resulting transfer function to the z domain by the bilinear transformation.

The work of Constantinides⁵ was used to develop the z domain transformations. The equations are given in appendix A.

⁵A. G. Constantinides, *Spectral Transformations for Digital Filters*, Proc. IEE, 117 (August 1970), 1585-1590.

3. FORTRAN IV PROGRAM

3.1 Outline

A FORTRAN IV computer program was written using the equations and the techniques discussed in section 2. The program was designed in a modular form to allow for easy modification as required. The flow chart of the program is given in figure 3. The main part of the program is simply a controller that calls in subroutines as required. The blocks in a column in the center of the flow chart are all in the main program, and the blocks to the right or the left of center are separate subroutines. For instance, if a different form of input data is required or a different output format is desired, then only the subroutine dealing specifically with that function need be modified. If a different analog design type or technique is to be used, it can be either substituted for one of the existing three design types or added on by extending a "computed go to" statement in the main program.

All data are stored in labeled and unlabeled common areas of a subroutine simply by including the proper common statement.

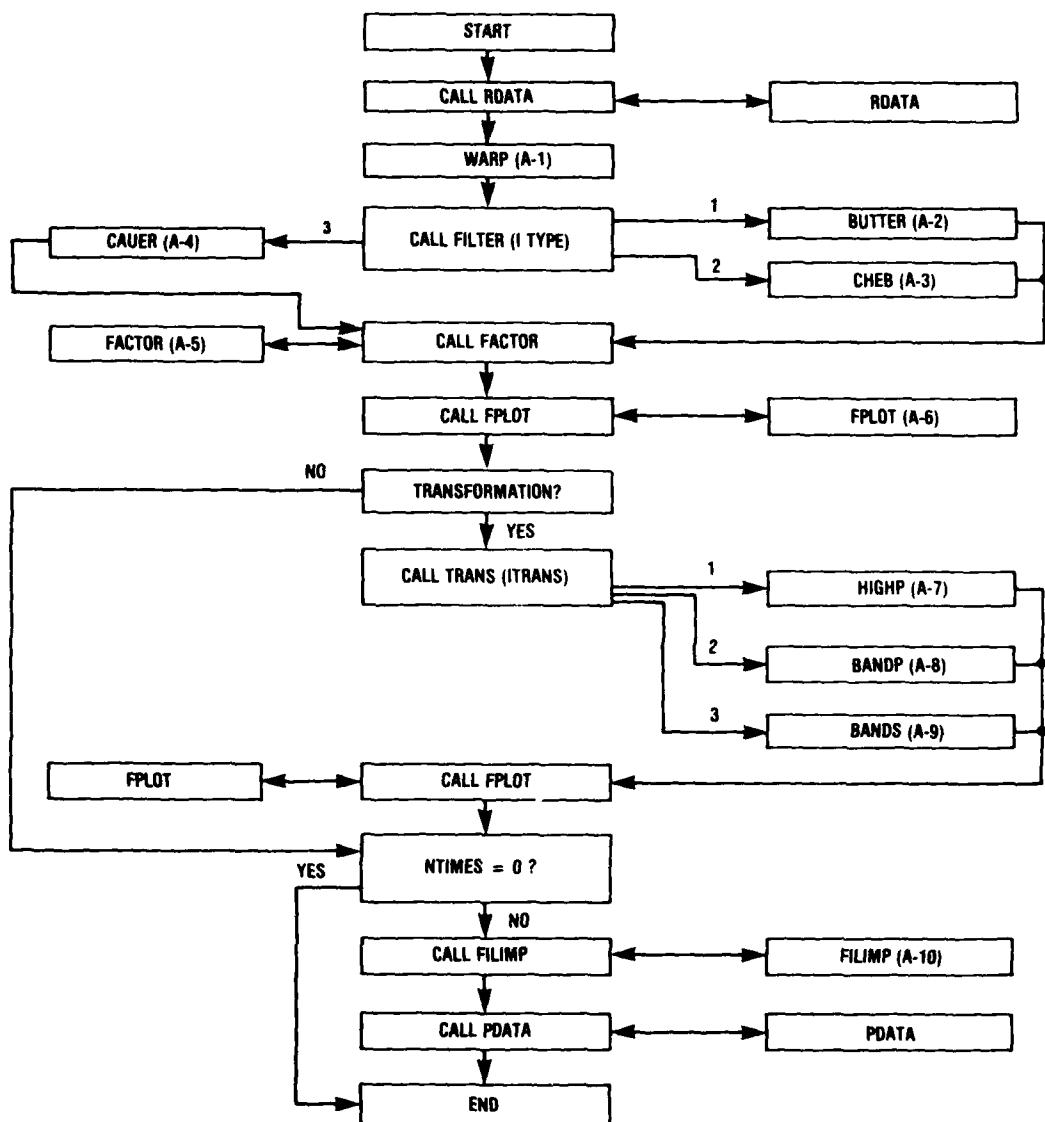
This program was developed by using an IBM System/370 computer. The H Extended Plus FORTRAN compiler was used with the auto-double option specified. This option doubles the overall precision specified in the program. Maximum possible precision is used for all the design calculations as well as in the implementation itself. This precision is 16 bytes for real variables and 32 bytes for complex variables. The complete program listing is given in appendix B.

The flow of the program is straightforward because it begins at the top and progresses without any diversions to the bottom of the chart. The basic steps of the program and the general function of each subroutine are as follows.

3.1.1 RDATA

After the program is initiated, it immediately calls the RDATA subroutine. This subroutine reads in the necessary filter design criteria and the data samples to be processed. Only batch processing of data is possible with the existing program, but the conversion to real time processing would not be difficult. The input data array is dimensioned for up to 1000 amplitude points at the specified sampling frequency. If only a filter design is required, then the number of data points (NTIMES) is set equal to zero, and RDATA bypasses the input of data to be processed. Setting NTIMES equal to zero bypasses also the implementation subroutine. RDATA also prints out the design criteria data for future reference.

The specific format required for data input is given in section 3.2.



NOTE: NUMBERS IN PARENTHESES CORRESPOND TO SECTIONS IN APPENDIX A.

Figure 3. Recursive digital filter program.

3.1.2 WARP

The RDATA subroutine returns control to the main program, which calculates the prewarped analog design frequency (WARP) and then calls the appropriate analog design subroutine. The analog filter subroutines calculate the poles and the zeros (if any) of the squared magnitude transfer function that lie in the left half of the complex s plane. The resulting transfer function after the right-half plane poles and zeros are eliminated is of the form

$$H(s) = \frac{(AMP)(s - z_1)(s - z_2) \dots (s - \bar{z}_1)}{(s - p_1)(s - p_2) \dots (s - \bar{p}_1)},$$

where \bar{z}_i and \bar{p}_i are complex conjugates of z_i and p_i (for all i). The equations needed to calculate these poles and zeros are given in appendix A and are derived as discussed in section 2.

3.1.3 FACTOR

The FACTOR subroutine is then called. FACTOR expands the transfer function into partial fractions. If the numbers of poles and zeros are equal, then this expansion has a constant gain equal to the amplitude multiplier (AMP) of the transfer function. FACTOR then combines the complex conjugate pairs and applies the bilinear transformation,

$$s = \frac{2}{T} \frac{1 - z^{-1}}{1 + z^{-1}},$$

to obtain the coefficients for the parallel second-order sections of the digital filter. The general form of the resulting z domain transfer function is⁶

$$H(z^{-1}) = (1 + z^{-1}) \sum_{i=1}^r \frac{A_{0i} + A_{1i}z^{-1}}{1 + B_{1i}z^{-1} + B_{2i}z^{-2}},$$

where $r = n/2$ for n even, $r = (n + 1)/2$ for n odd, and n is the order of the filter. The coefficients A_{0i} , A_{1i} , B_{1i} , and B_{2i} are calculated by FACTOR using the analog poles and zeros. If the order of the filter is odd, then there is one real pole, so the A_1 and B_2 coefficients are equal to zero for that pole.

⁶G. C. Temes and S. K. Mitra, *Modern Filter Theory and Design*, John Wiley & Sons, Inc., New York (1973).

3.1.4 F PLOT

A plot of the frequency response (magnitude and phase) of the low-pass digital filter is then made by the F PLOT subroutine. It is plotted by letting

$$z^{-1} = e^{-j\omega T} = \cos \omega T - j \sin \omega T$$

in the z domain transfer function given above.

The plots given in this report were made on a Houston Instruments plotter using existing plotting software.⁷ Both the amplitude and phase responses are plotted over the normalized range of frequencies of zero to one. That is, the frequency scale is normalized to the sampling frequency. F PLOT also prints the values of the digital filter coefficients to 25 decimal places for future use.

3.1.5 Filter Choice

The main program then determines if a high-pass, band-pass, or band-stop filter is desired or if the low-pass filter is ready to be implemented.

The transfer function for the required high-pass, band-pass, or band-stop filter is obtained by an appropriate transformation in the z domain of the low-pass digital filter. Band-pass and band-stop transformations result in a doubling of the filter order.

After transformation, the frequency characteristics of the new filter are plotted by F PLOT.

3.1.6 FILIMP

The actual implementation of the digital filter is simple. The FILIMP subroutine performs this function and processes the data to be filtered. Application of the inverse z transform to the general form for the z domain transfer function results in the recursive difference equation

$$Y_{out}(kT) = X(kT) + X[(k - 1)T] ,$$

⁷Thomas V. Noon and Egon Marx, User's Manual for the Modular Analysis-Package Libraries ANAPAC and TRANL, Harry Diamond Laboratories HDL-TR-1782-S (September 1978).

where

$$X(kT) = \sum_{i=1}^r X_i(kT) ,$$

$$X_i(kT) = A_{0i}Y_{in}(kT) + A_{1i}Y_{in}[(k-1)T] \\ - B_{1i}X_i[(k-1)T] - B_{2i}X_i[(k-2)T] .$$

3.1.7 PDATA

The PDATA subroutine provides plots of the input and output data. This routine may be easily modified to provide the output data in any desired format.

3.2 Program Use

There are at most nine parameters that must be supplied to the program for the design of a filter. The minimum number needed is five for the design of a Butterworth low-pass filter. This minimum set, which is used in all designs, consists of these parameters:

- a. ITYPE, the type of analog design

1 = Butterworth

2 = Chebyshev

3 = Elliptic (Cauer)

- b. N, the order of the filter

- c. ITRANS, corresponding to the band transformation desired

0 = None

1 = High-pass transformation

2 = Band-pass transformation

3 = Band-stop transformation

- d. FC, the cutoff frequency desired (in hertz)

- e. FS, the sampling frequency of the data (in hertz) to which the filter will be normalized

If a low-pass Chebyshev is required, then an additional parameter, EPS1, the minimum allowed amplitude in the passband, also must be entered. Similarly, for elliptic filters, EPS1 must be given along with EPS2, the transition region selectivity factor. The maximum value for EPS2 is 0.95. All of these parameters are read from the same data card. These are the formats for all the data cards and the order in which the cards are read by the program:

ITYPE,N,ITRANS,FC,FS,EPS1,EPS2
Format--3I5, 4E10.3

If ITRANS = 0, skip reading F1 and F2.

F1,F2
Format--2E10.3

NTIMES
Format--I5

If NTIMES = 0, skip reading VIN.

VIN
Format--8E10.3--maximum of 125 cards

Heading for low-pass design

If ITRANS = 0, skip next heading.

Heading for band transformation

If NTIMES = 0, skip next two headings.

Heading for VIN plot

Heading for VOUT plot

If a band transformation is specified, then an additional data card is needed that has the lower cutoff frequency, F1 (also the high-pass cutoff frequency), and the upper cutoff frequency, F2, for the band-pass and band-stop filters.

Next, the number of data sample points is read. If NTIMES is set equal to zero, then no input data are read, and the program is terminated after completion of the filter design.

Separate titles are read in for the low-pass filter transfer function plots, any band transformation frequency plots, and the heading to appear on the data plots. These titles can be up to 80 characters of any form desired.

A sample filter design is given now to illustrate the order and the manner in which the data are assembled. A fourth-order elliptic band-pass filter is chosen since this design requires input of all the design parameters. We have this:

ITYPE = 3

N = 4

ITRANS = 2

Now let us assume that the sampling frequency is 10,000 Hz and that the passband is to be from 1000 to 3000 Hz. In addition, we allow the passband amplitude to vary between 1.0 and 0.9 and minimize the transition between the passband and the stop band. This action means that the cutoff frequency for the low-pass design must be 2000 Hz, which is the width of the passband. Therefore, the remaining parameters are these:

FC = 2000.0

FS = 10000.0

EPS1 = 0.9

EPS2 = 0.95

F1 = 1000.0

F2 = 3000.0

These data are given in figure 4 as they appear on the input data cards. The plot title cards also are shown in the figure. If only a low-pass design is requested, then the second title card is omitted. Similarly, if no data points are given, then the third title card as well as the input data cards are omitted.

The resulting printout for this example problem is given in figure 5. The low-pass transfer function is given in figures 6 and 7, and the band-pass function is given in figures 8 and 9. Running this example required 6.85 s in the central processing unit (CPU) to compile the FORTRAN coding, 2.44 s in the CPU to link and edit the object modules with library functions, and about 2 s in the CPU to perform the actual calculations. The filtering of 101 data points required less than 2 s of CPU time. Additional examples of filter designs are given in section 4.

HEADING FOR OUTPUT DATA PLOT
HEADING FPP INPUT DATA PLOT
HEADING FOR BANDPASS DESIGN
HEADING FOR LOWPASS DESIGN

2.0E0	1.0E0	0.0	-1.0E0	-1.5E0	-1.0E0	-3.0E-1	-2.5E-1
0.0	1.0E0	2.0E0	3.0E0	4.0E0	5.0E0	4.0E0	3.0E0
16							
1.0E3	3.0E3						
3	4	2	2.0E3	1.0E4	0.9	0.95	
1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9
0	0	0	0	0	0	0	0

Figure 4. Input data for example 1.

Figure 5. Output data for example 1.

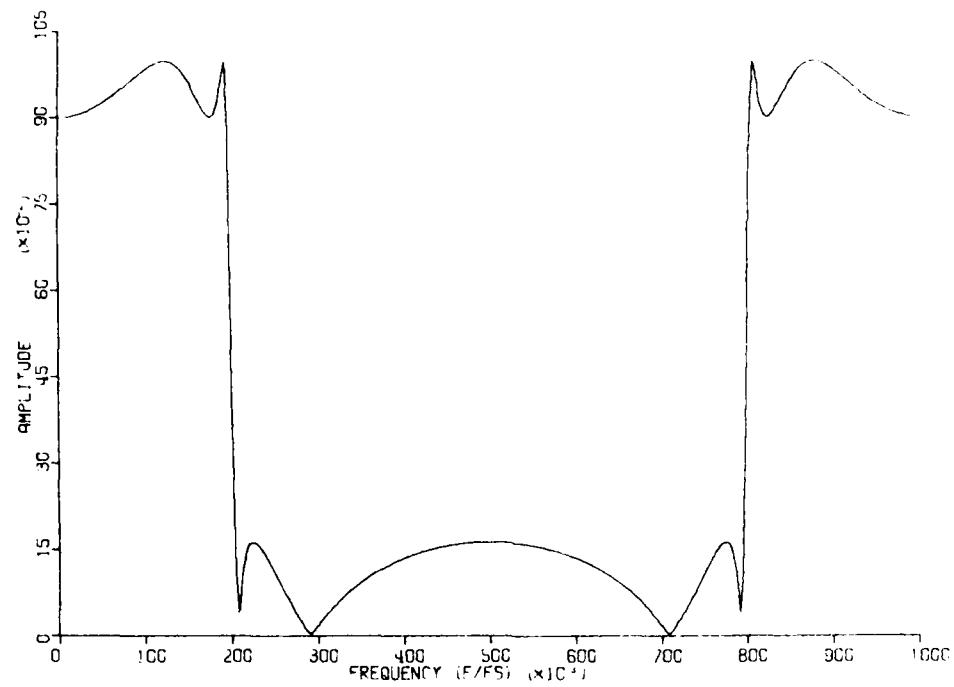


Figure 6. Low-pass Cauer filter: amplitude versus frequency.

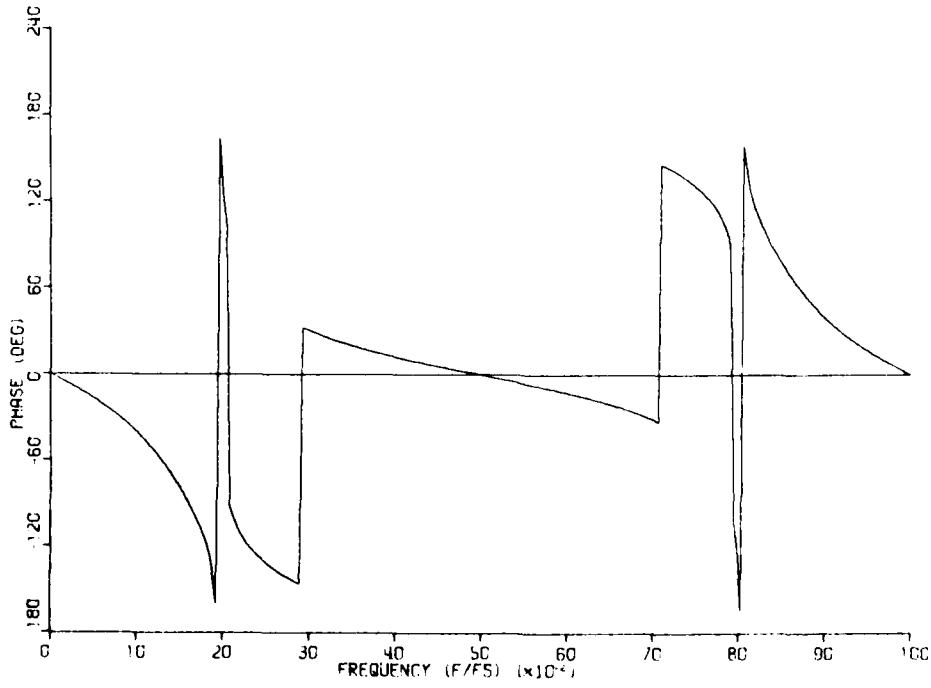


Figure 7. Low-pass Cauer filter: phase versus frequency.

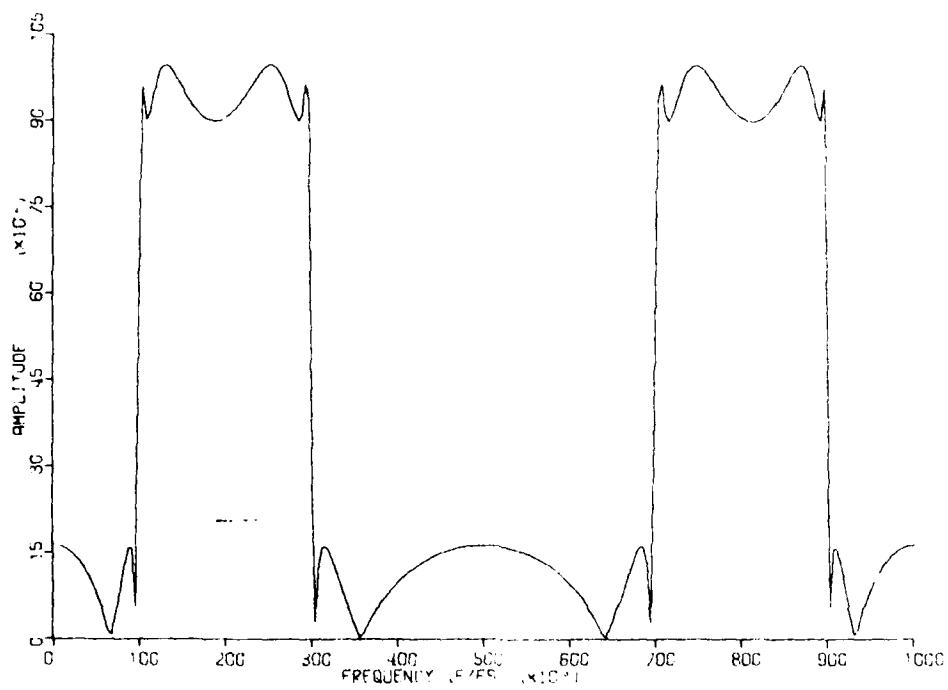


Figure 8. Band-pass Cauer filter: amplitude versus frequency.

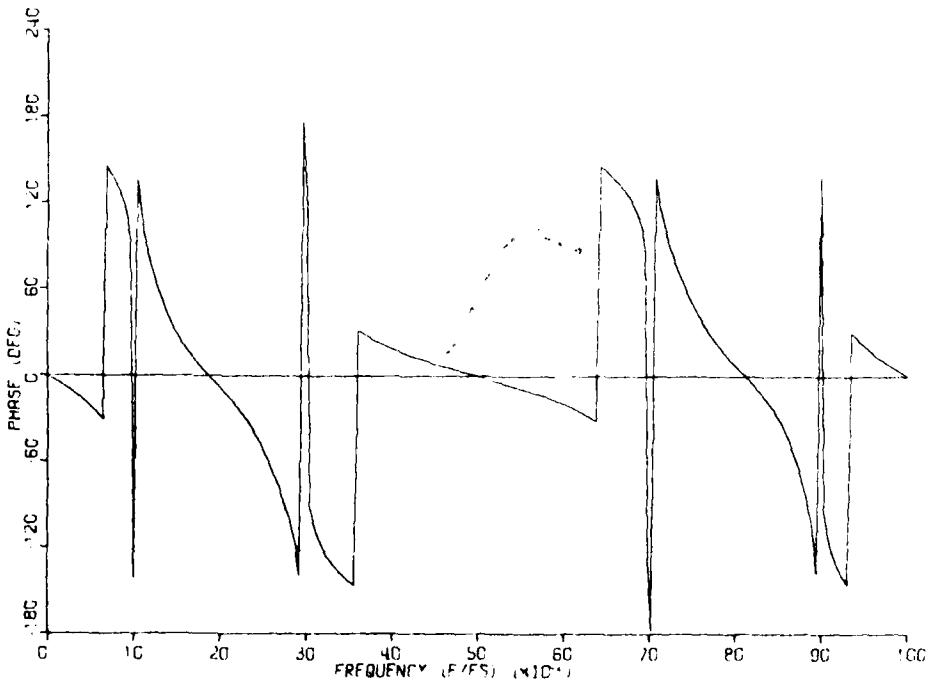


Figure 9. Band-pass Cauer filter: phase versus frequency.

4. DESIGN EXAMPLES

Several examples are given in this section to illustrate the differences in the various analog designs and also to show the results of band transformations. In addition, a simple waveform consisting of three sinusoids at different frequencies is used to demonstrate the filtering action of the various digital filters.

4.1 Analog Design

The three types of analog designs are discussed in section 2. A graphical presentation of the basic differences in these three designs is given here. A fourth-order elliptic low-pass design is given in section 3.2. Similar fourth-order digital filters using the Butterworth and Chebyshev analog designs are presented in figures 10 to 13. Comparing these transfer functions, we can easily see that the Butterworth filter offers the smoothest response in both the passband and the stop band. The sharper transition from passband to stop band of the Chebyshev design over the Butterworth is obtained at the expense of smoothness in the passband. The elliptic design obviously has the sharpest transition of the three designs. However, the ripple in both the passband and the stop band is the penalty for this sharp transition. The design to use is chosen according to the requirements of the specific filtering problem being considered.

4.2 Band Transformations

An example of a band-pass transformation is given in figures 8 and 9. The low-pass Butterworth filter of figures 10 and 11 was transformed to a high-pass filter with a cutoff of 3000 Hz as shown in figures 14 and 15. The band-stop dual of the passband filter was made by using the Chebyshev design and is shown in figures 16 and 17.

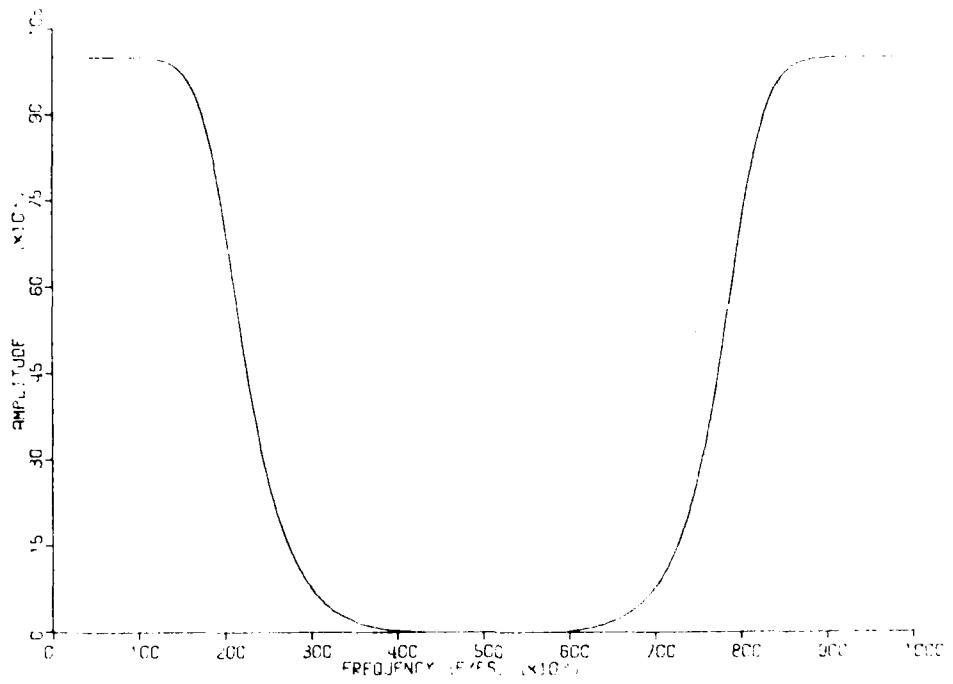


Figure 10. Low-pass Butterworth filter: amplitude versus frequency.

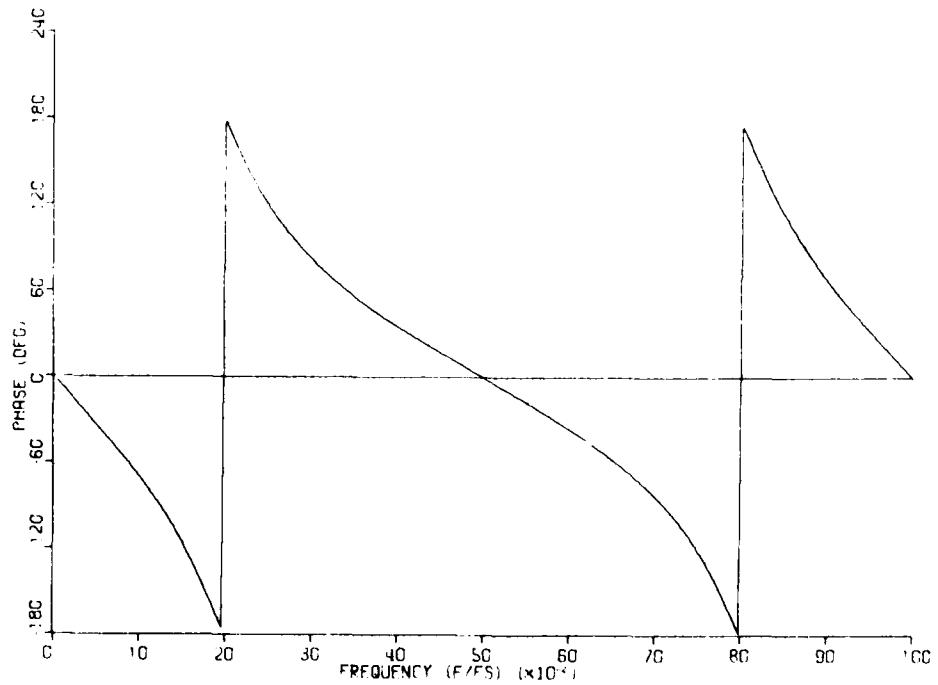


Figure 11. Low-pass Butterworth filter: phase versus frequency.

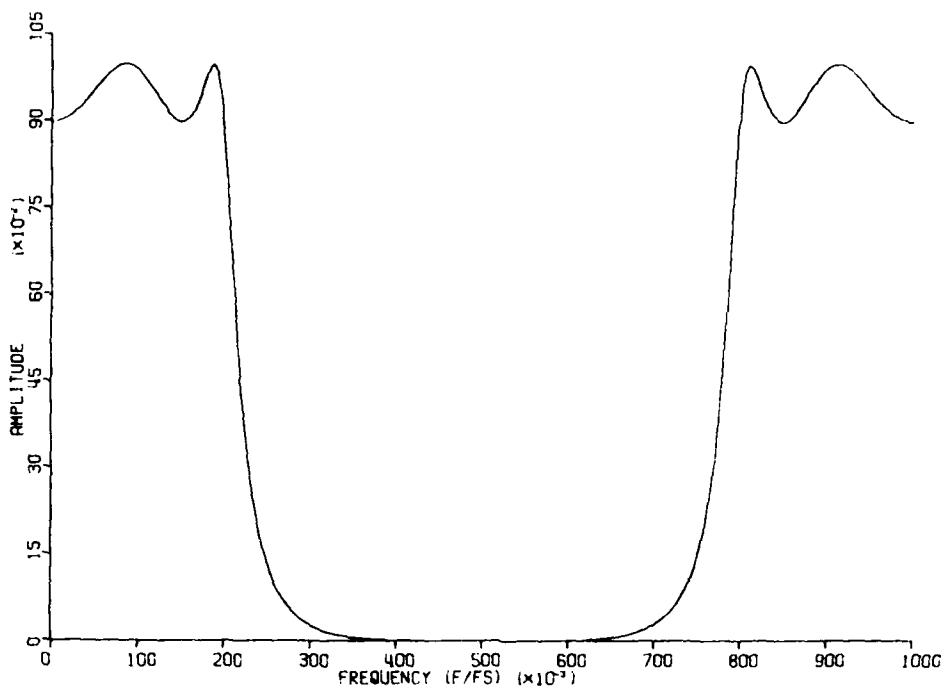


Figure 12. Low-pass Chebyshev filter: amplitude versus frequency.

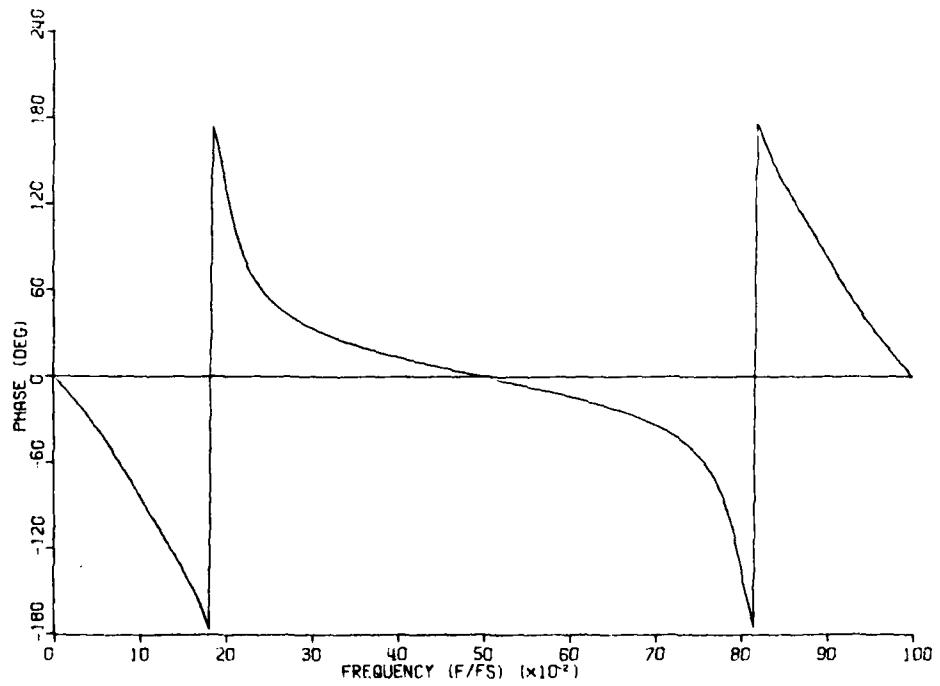


Figure 13. Low-pass Chebyshev filter: phase versus frequency.

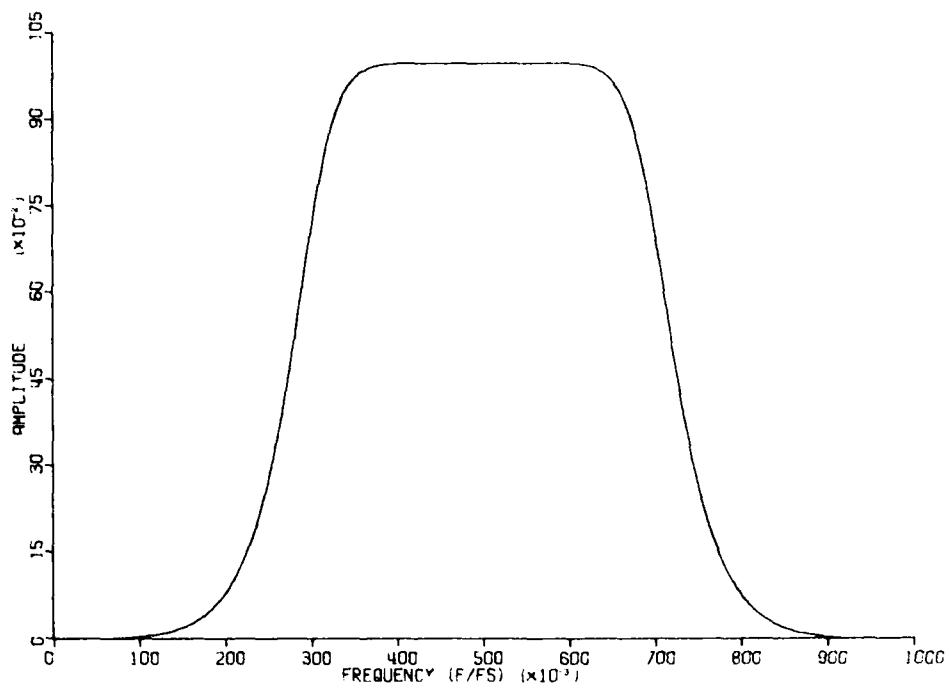


Figure 14. High-pass Butterworth filter: amplitude versus frequency

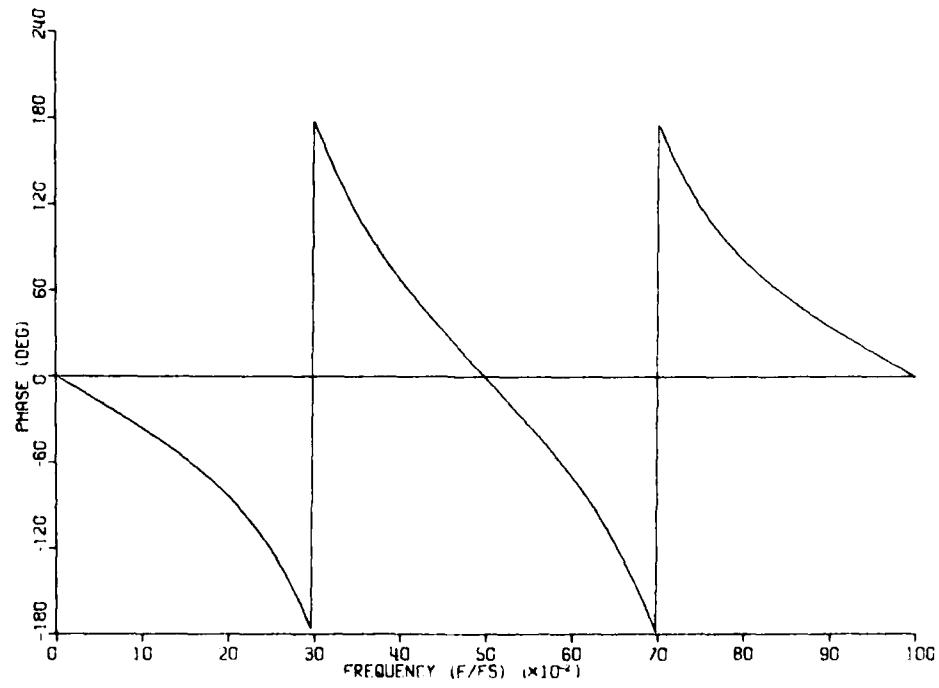


Figure 15. High-pass Butterworth filter: phase versus frequency.

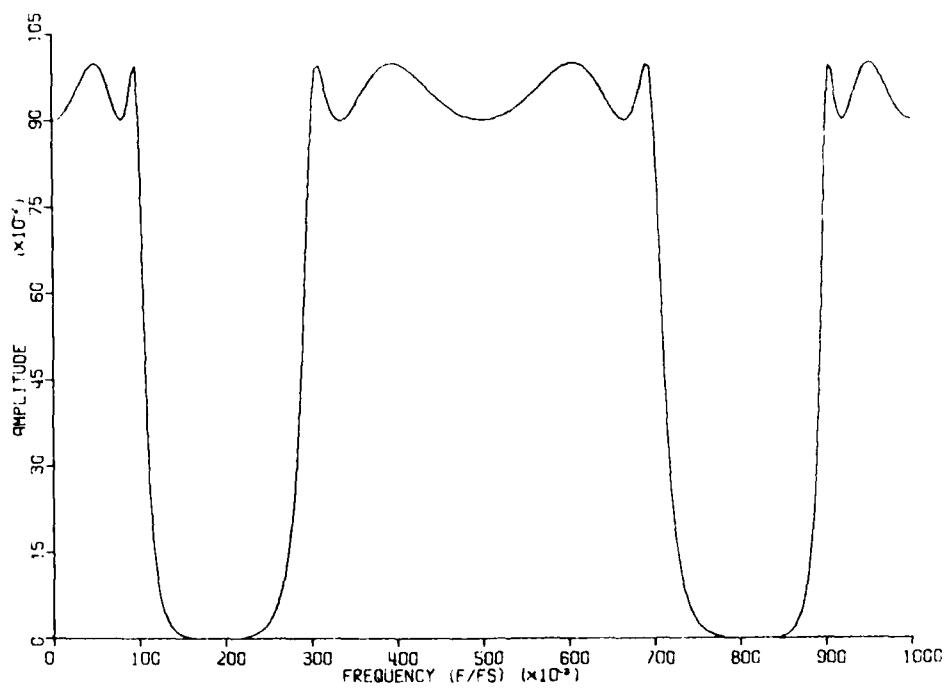


Figure 16. Band-stop Chebyshev filter: amplitude versus frequency.

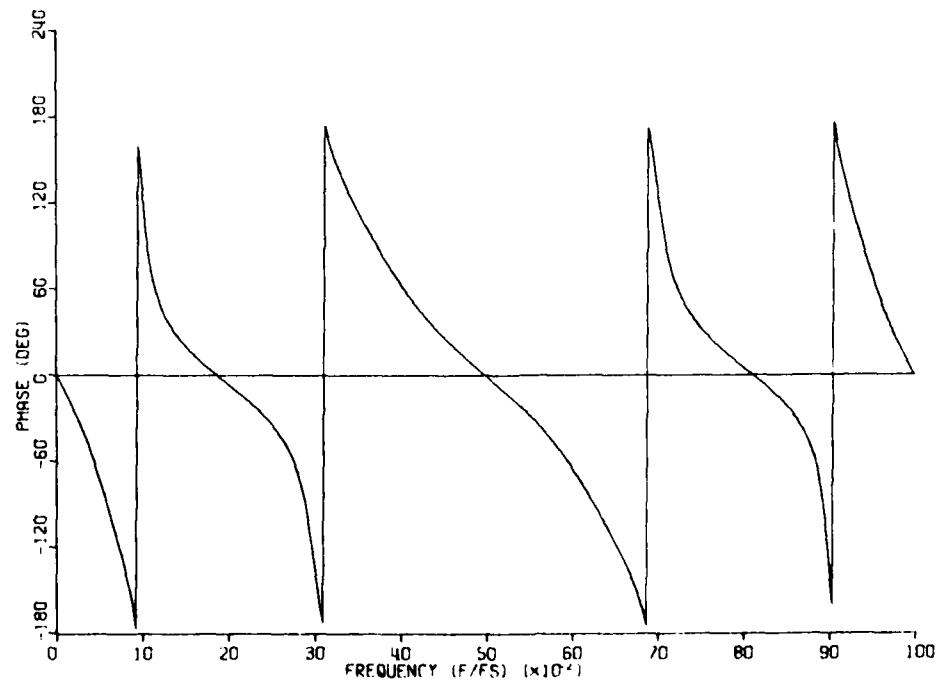


Figure 17. Band-stop Chebyshev filter: phase versus frequency.

4.3 Data Processing

A simple waveform was generated to illustrate the operation of the filters on input data. The waveform used was the sum of three sinusoids at 0.05, 0.15, and 0.35 times the sampling frequency with peak amplitudes of 3, 2, and 5, respectively. This input waveform is shown in figure 18. A 20th-order Butterworth low-pass design was used to eliminate the two highest frequency sine waves, as shown in figure 19. A filter of high order was used to demonstrate the group delay effect of the filter. This effect shows at the beginning of the output waveform as a delay time before any output occurs. Next, a fourth-order Butterworth high-pass filter was used to eliminate the two low-frequency signals. This result is shown in figure 20. The low sampling rate caused the distortion on the sine wave seen in this figure. The initial design example was used to pass only the center sinusoid as shown in figure 21. Here, again, the effect of the low sampling rate is seen as a distortion of the waveform. However, the effect is not as great because of the lower frequency of the sinusoid. The Chebyshev band-stop dual of this passband filter was then used to eliminate the middle sine wave. This waveform is given in figure 22.

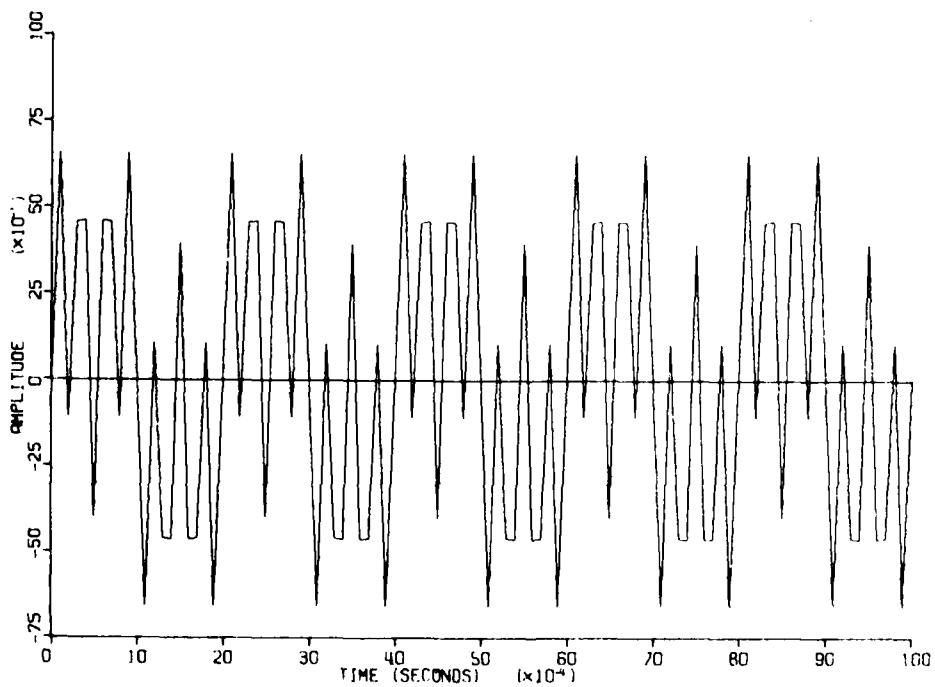


Figure 18. Input waveform obtained by summing three sinusoids.

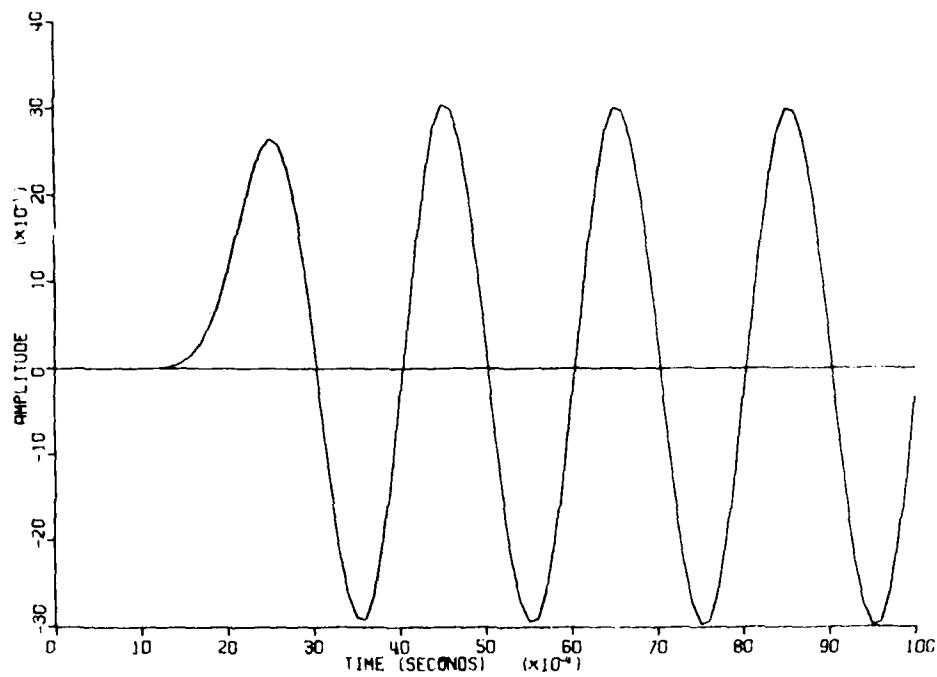


Figure 19. Output waveform obtained by using low-pass Butterworth filter.

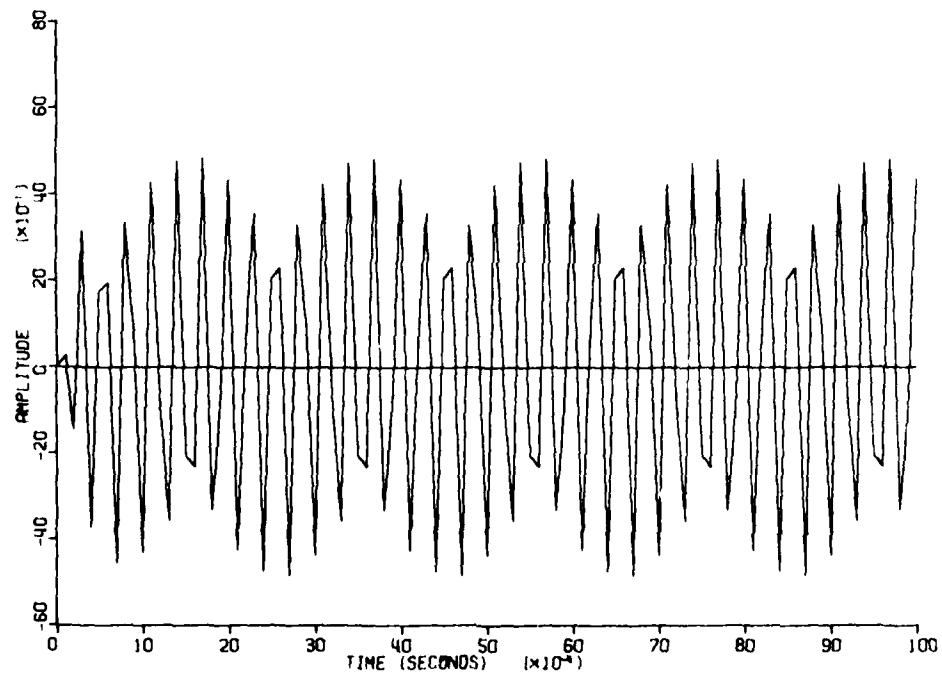


Figure 20. Output waveform obtained by using high-pass Butterworth filter.

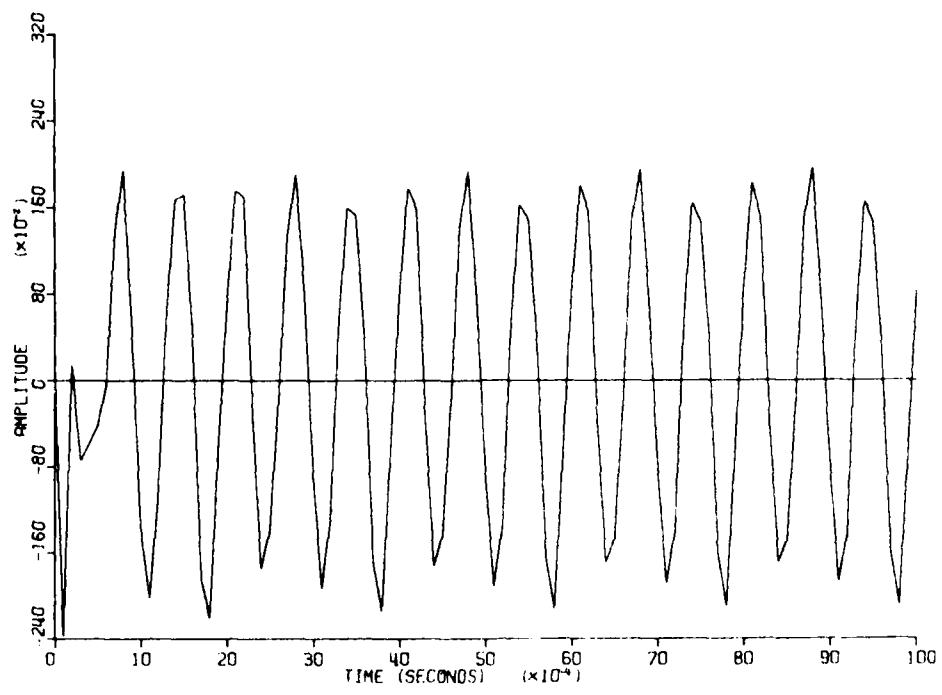


Figure 21. Output waveform obtained by using band-pass Cauer filter.

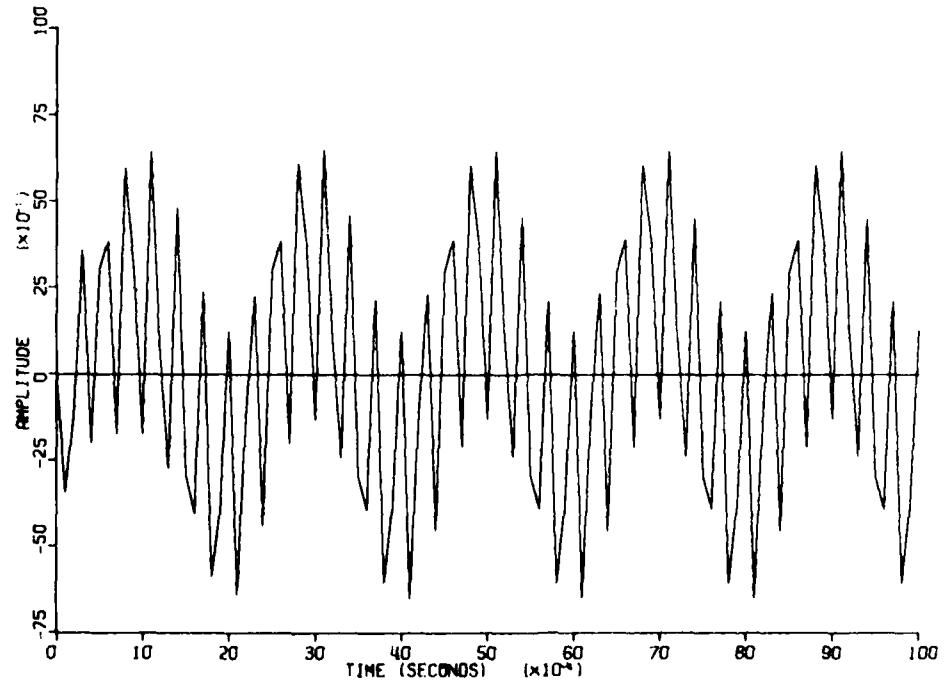


Figure 22. Output waveform obtained by using band-stop Chebyshev filter.

5. CONCLUSIONS AND RECOMMENDATIONS

A versatile program has been developed for designing and implementing recursive digital filters. Three well-known analog filter designs were used with the bilinear transformation to obtain the desired digital filters. The bilinear technique was chosen because it eliminates the effects of aliasing that occur with other techniques. However, the resulting warping of the frequency scale may not be acceptable in some applications. The extent to which this warping affects the resulting output was not investigated. Examples using the different analog designs are given in section 4 along with transformations from low-pass to high-pass, band-pass, and band-stop designs. All of the filter design calculations as well as the implementations use the highest precision possible for the IBM System/370 computer. No investigation was made to determine the effects of using lower precision.

All of the equations used in developing the code are included in appendix A, and the complete computer program code is listed in appendix B. A bibliography of related publications also is included. This bibliography is by no means complete since the amount of information published seems almost endless. Although there is a large amount of information available, no one source proved adequate in presenting all of the steps necessary to completely develop a digital filter. It is hoped, therefore, that this document will prove helpful to others venturing into this field for the first time.

Although the program developed here is completely operable, there are many ways in which the program may be improved or modified to better suit a particular filtering need. Also, investigation and analysis of the various errors associated with the techniques used should be conducted.

LITERATURE CITED

- (1) A. Antoniou, Digital Filters: Analysis and Design, McGraw-Hill Book Co., Inc., New York (1979).
- (2) M. S. Ghausi, Principles and Design of Linear Active Circuits, McGraw-Hill Book Co., Inc., New York (1965), ch. 4.
- (3) H. Y.-F. Lam, Analog and Digital Filters, Prentice-Hall, Inc., Englewood Cliffs, NJ (1979).
- (4) D. E. Johnson, Introduction to Filter Theory, Prentice-Hall, Inc., Englewood Cliffs, NJ (1976).
- (5) A. G. Constantinides, Spectral Transformations for Digital Filters, Proc. IEE, 117 (August 1970), 1585-1590.
- (6) G. C. Temes and S. K. Mitra, Modern Filter Theory and Design, John Wiley & Sons, Inc., New York (1973).
- (7) Thomas V. Noon and Egon Marx, User's Manual for the Modular Analysis-Package Libraries ANAPAC and TRANL, Harry Diamond Laboratories HDL-TR-1782-S (September 1978).

SELECTED BIBLIOGRAPHY

Blinchikoff, H. J., and Zverev, A. I., Filter in the Time and Frequency Domains, John Wiley & Sons, Inc., New York (1976).

Cappellini, V., Constantinides, A. G., and Emiliani, P., Digital Filters and Their Applications, Academic Press, Inc., New York (1978).

Childers, D., and Durling, A., Digital Filtering and Signal Processing, West Publishing Co., St. Paul, MN (1975).

Darlington, S., Simple Algorithms for Elliptic Filters and Generalizations Thereof, IEEE Trans. Circuits Syst., CAS-25 (December 1978), 975-980.

Deczky, A. G., Equiripple and Minimax (Chebyshev) Approximations for Recursive Digital Filters, IEEE Trans. Acoust., Speech, Signal Process., ASSP-22 (April 1974).

Fettter, S. A., Introduction to Discrete-Time Signal Processing, John Wiley & Sons, Inc., New York (1976).

Gibbs, A. J., The Design of Digital Filters, Aust. Telecommun. Res. J., 4 (1970), 29-34.

Gold, B., and Rader, C. M., Digital Processing of Signals, McGraw-Hill Book Co., Inc., New York (1969).

Hamming, R. W., Digital Filters, Prentice-Hall, Inc., Englewood Cliffs, NJ (1977).

Liu, B., Digital Filters and the Fast Fourier Transform, Halsted Press, New York (1975).

Oppenheim, A. V., Applications of Digital Signal Processing, Prentice-Hall, Inc., Englewood Cliffs, NJ (1978).

Oppenheim, A. V. and Schafer, R. W., Digital Signal Processing, Prentice-Hall, Inc., Englewood Cliffs, NJ (1975).

Rader, C. M., and Gold, B., Digital Filter Design Techniques in the Frequency Domain, Proc. IEEE, 55 (February 1967), 149-171.

Rakovitch, B. D., and Litovski, V. B., Monotonic Passband Low-Pass Filters with Chebyshev Stopband Attenuation, IEEE Trans. Acoust., Speech, Signal Process., ASSP-22 (February 1974).

Stearns, S. D., Digital Signal Processing, Hayden Book Co., Inc., Rochelle Park, NJ (1975).

APPENDIX A.--EQUATIONS USED FOR CODE DEVELOPMENT

CONTENTS

	<u>Page</u>
A-1. PREWARPING	37
A-2. BUTTERWORTH	37
A-3. CHEBYSHEV	37
A-4. ELLIPTIC	39
A-5. FACTOR	41
A-6. MAGNITUDE AND PHASE	42
A-7. HIGH-PASS TRANSFORMATION	43
A-8. BAND-PASS TRANSFORMATION	43
A-9. BAND-STOP TRANSFORMATION	45
A-10. FILTER IMPLEMENTATION	46

APPENDIX A

The following equations were used in the development of the computer code for designing and implementing digital filters. All of the equations are referenced in the main body of this report. The application of each set of equations is given in the flow chart of the program (fig. 3).

A-1. PREWARPING (WARP)

$$\tilde{f}_c = \frac{f_{AC}}{f_s} = \frac{1}{\pi} \tan \left(\pi \frac{f_{DC}}{f_s} \right).$$

A-2. BUTTERWORTH

The transfer function for a Butterworth low-pass filter of order n is given by

$$H(s) = \frac{H_0}{\prod_{k=1}^n (s - p_k)},$$

where

$$H_0 = \tilde{\omega}_c^n,$$

$\tilde{\omega}_c$ = cutoff frequency,

$$p_k = \tilde{\omega}_c e^{j[(\pi/2)+\pi(2k-1)/2n]}, \quad k = 1, 2, \dots, n.$$

A-3. CHEBYSHEV

The squared magnitude function is of the form

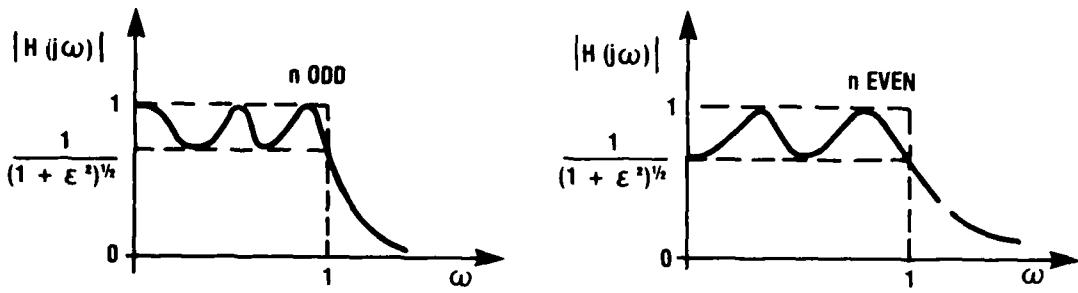
$$|H(j\omega)|^2 = \frac{1}{1 + \epsilon^2 C_n^2(\omega)},$$

where

ϵ = ripple factor,

$C_n(\omega)$ = Chebyshev polynomial.

APPENDIX A



For the program, ϵ is entered as

$$\text{EPS1} = 1 - \frac{1}{(1 + \epsilon^2)^{1/2}} .$$

The transfer function is of the form

$$H(s) = \frac{H_0}{\prod_{k=1}^n (s - p_k)} ,$$

where

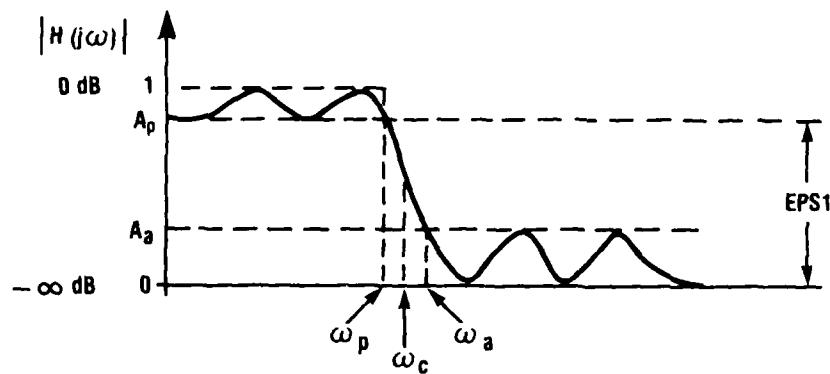
$$p_k = \sin(u_k) \sinh(v) + j \cos(u_k) \cosh(v) = \alpha_k + j\beta_k ,$$

$$u_k = \frac{\pi}{2n} (2k - 1), \quad k = n + 1, n + 2, \dots, 2n ,$$

$$v = \frac{1}{n} \sinh^{-1} \left(\frac{1}{\epsilon} \right) ,$$

$$H_0 = \begin{cases} [a_{(n+1)/2}] \prod_{k=1}^{(n-1)/2} (\alpha_k^2 + \beta_k^2), & n \text{ odd}, \\ \frac{1}{(1 + \epsilon^2)^{1/2}} \prod_{k=1}^{n/2} (\alpha_k^2 + \beta_k^2), & n \text{ even}. \end{cases}$$

A-4. ELLIPTIC (CAUER)



For the normalized filter $\omega_c = 1$,

n = filter order,

$$\omega_c = (\omega_p \omega_a)^{1/2} = 1,$$

k = selectivity factor $= \omega_p / \omega_a$,

A_p = maximum passband attenuation (in decibels),

A_a = minimum stop-band attenuation (in decibels).

Of the parameters A_p , A_a , k , and n , only three are independent. If three of the four are specified, the fourth is automatically fixed. For this project, n , k , and A_p are specified:

$k = \text{EPS2}$,

$$A_p = 20 \log (1 - \text{EPS1}).$$

A_a is then given by

$$A_a = 10 \log \left(\frac{10^{-0.1A_p} - 1}{16q^n} + 1 \right).$$

APPENDIX A

The normalized elliptic low-pass filter has a transfer function of the form

$$H_N(s) = \frac{H_0}{D_0(s)} \prod_{i=1}^r \frac{s^2 + A_{0i}}{s^2 + B_{1i}s + B_{0i}},$$

where

$$r = \begin{cases} (n-1)/2, & n \text{ odd}, \\ n/2, & n \text{ even}, \end{cases}$$

$$D_0(s) = \begin{cases} s + \sigma_0, & n \text{ odd}, \\ 1, & n \text{ even}. \end{cases}$$

The transfer function coefficients and multiplier constant H_0 are computed by using the following formulas in sequence:

$$k' = (1 - k^2)^{1/2}, \quad (A-1)$$

$$q_0 = \frac{1}{2} \frac{1 - \sqrt{k'}}{1 + \sqrt{k'}}, \quad (A-2)$$

$$q = q_0 + 2q_0^5 + 15q_0^9 + 150q_0^{13}, \quad (A-3)$$

$$\Lambda = \frac{1}{2n} \ln \left(\frac{\frac{-0.05A_p}{10} + 1}{\frac{-0.05A_p}{10} - 1} \right), \quad (A-4)$$

$$\sigma_0 = \left| \frac{2q^{1/4} \sum_{m=0}^{\infty} (-1)^m q^{m(m+1)} \sinh [(2m+1)\Lambda]}{1 + 2 \sum_{m=1}^{\infty} (-1)^m q^{m^2} \cosh (2m\Lambda)} \right|, \quad (A-5)$$

$$\Omega_i = \frac{2q^{1/4} \sum_{m=0}^{\infty} (-1)^m q^{m(m+1)} \sin \left[\frac{(2m+1)\pi u}{n} \right]}{1 + 2 \sum_{m=1}^{\infty} (-1)^m q^{m^2} \cos \left(\frac{2m\pi u}{n} \right)}, \quad (A-6)$$

APPENDIX A

where

$$\mu = \begin{cases} i, & n \text{ odd}, \\ i - \frac{1}{2}, & n \text{ even}, \end{cases} \quad i = 1, 2, \dots, r.$$

The series in equations (A-5) and (A-6) converge rapidly, and usually three or four terms are sufficient.

$$w = \left[(1 + k\sigma_0^2)(1 + \sigma_0^2/k) \right]^{1/2}, \quad (A-7)$$

$$v_i = \left[(1 - k\Omega_i^2)(1 - \Omega_i^2/k) \right]^{1/2}, \quad (A-8)$$

$$A_{0i} = 1/\Omega_i^2, \quad (A-9)$$

$$B_{0i} = \frac{(\sigma_0 v_i)^2 + (\Omega_i w)^2}{(1 + \sigma_0^2 \Omega_i^2)^2}, \quad (A-10)$$

$$B_{1i} = \frac{2\sigma_0 v_i}{1 + \sigma_0^2 \Omega_i^2}, \quad (A-11)$$

$$H_0 = \begin{cases} \sigma_0 \prod_{i=1}^r \frac{B_{0i}}{A_{0i}}, & n \text{ odd}, \\ 10^{-0.05A_p} \prod_{i=1}^r \frac{B_{0i}}{A_{0i}}, & n \text{ even}. \end{cases} \quad (A-12)$$

A-5. FACTOR

$$R_k + jI_k = \frac{H_0 \prod_{m=1}^{n'} (p_k - z_m)}{\prod_{\ell=1}^n (p_k - p_\ell)}, \quad \ell \neq k, \quad n' = \begin{cases} n, & n \text{ even}, \\ n-1, & n \text{ odd}, \end{cases}$$

APPENDIX A

$$H(s) = \sum_{k=1}^r \left(\frac{R_k + jI_k}{s - p_k} + \frac{R_k - jI_k}{s - \bar{p}_k} \right), \quad r = \begin{cases} (n+1)/2, & n \text{ odd}, \\ n/2, & n \text{ even}, \end{cases}$$

$$s = \frac{2}{T} \frac{1 - z^{-1}}{1 + z^{-1}},$$

$$H(z^{-1}) = (1 + z^{-1}) \sum_{k=1}^r \frac{A_{0k} + A_{1k}z^{-1}}{1 + B_{1k}z^{-1} + B_{2k}z^{-2}},$$

$$p_k = -\alpha_k + j\beta_k,$$

$$A_{0k} = [R_k(1 + \alpha_k/2) - I_k(\beta_k/2)]/D,$$

$$A_{1k} = -[R_k(1 - \alpha_k/2) + I_k(\beta_k/2)]/D,$$

$$B_{1k} = -2[1 - (\alpha_k/2)^2 - (\beta_k/2)^2]/D,$$

$$B_{2k} = [(\alpha_k/2)^2 + (\beta_k/2)^2]/D,$$

$$D = (1 + \alpha_k/2)^2 + (\beta_k/2)^2.$$

A-6. MAGNITUDE AND PHASE (FPLOT)

The frequency response of the designed digital filter is calculated by substituting $e^{-j\omega T}$ for z^{-1} in the transfer function and then using complex arithmetic to obtain the magnitude and the phase as a function of frequency.

$$z^{-1} + e^{-j\omega T} = w = \cos \omega T - j \sin \omega T,$$

$$H(w) = (1 + w) \sum_{k=1}^r \frac{A_{0k} + A_{1k}w}{1 + B_{1k}w + B_{2k}w^2}.$$

APPENDIX A

A-7. HIGH-PASS TRANSFORMATION

A low-pass digital filter of cutoff frequency θ_p is first designed for the desired order, n, and type. Let ω_p = the desired high-pass cutoff frequency.

Then substituting for z^{-1} in the low-pass transfer function,

$$z^{-1} \rightarrow -\frac{z^{-1} + \alpha}{1 + \alpha z^{-1}} ,$$

where

$$\alpha = -\frac{\cos\left(\frac{\omega_p + \theta_p}{2}\right)}{\cos\left(\frac{\omega_p - \theta_p}{2}\right)},$$

results in the transfer function of the desired high-pass filter.

$$H_{HP}(z^{-1}) = (1 - z^{-1}) \sum_{k=1}^r \frac{A'_{0k} + A'_{1k}z^{-1}}{1 + B'_{1k}z^{-1} + B'_{2k}z^{-2}} ,$$

$$A'_{0k} = (1 - \alpha) \left(\frac{A_{0k} - \alpha A_{1k}}{D'} \right) ,$$

$$A'_{1k} = (1 - \alpha) \left(\frac{\alpha A_{0k} - A_{1k}}{D'} \right) ,$$

$$B'_{1k} = [2\alpha - B_{1k}(1 + \alpha^2) + 2B_{2k}\alpha]/D' ,$$

$$B'_{2k} = (\alpha^2 - \alpha B_{1k} + B_{2k})/D' ,$$

$$D' = 1 - \alpha B_{1k} + \alpha^2 B_{2k} .$$

A-8. BAND-PASS TRANSFORMATION

A low-pass digital filter of cutoff frequency β is first designed for the desired order and type. The cutoff frequency equals the bandwidth of the desired band-pass filter, $F_2 - F_1$, where F_2 and F_1 are the upper and lower cutoff frequencies (in hertz), respectively.

APPENDIX A

A substitution for z^{-1} in the low-pass transfer function results in the transfer function of the desired band-pass filter.

In general, this transformation is^{1,2}

$$z^{-1} \rightarrow -\frac{z^{-2} - \frac{2\alpha k}{k+1} z^{-1} + \frac{k-1}{k+1}}{\frac{k-1}{k+1} z^{-2} - \frac{2\alpha k}{k+1} z^{-1} + 1},$$

where

$$\alpha = \frac{\cos\pi(F_1 + F_2)T}{\cos\pi(F_2 - F_1)T},$$

$$k = \cot[\pi(F_2 - F_1)T] \tan(\pi\beta T).$$

By letting $k = 1$, then $\beta = F_2 - F_1$, and the transformation is simplified.

$$z^{-1} \rightarrow -z^{-1} \left(\frac{z^{-1} - \alpha}{1 - \alpha z^{-1}} \right).$$

The resulting band-pass transfer function has the form

$$H_{BP}(z^{-1}) = (1 - z^{-2}) \sum_{k=1}^{n/2} \left(\frac{A'_{11k}z^{-1} + A'_{01k}}{1 + B'_{11k}z^{-1} + B'_{21k}z^{-2}} - \frac{A'_{12k}z^{-1} - A'_{02k}}{1 - B'_{12k}z^{-1} + B'_{22k}z^{-2}} \right)$$

¹A. G. Constantinides, *Spectral Transformations for Digital Filters*, Proc. IEE, 117 (August 1970), 1585-1590.

²A. Antoniou, *Digital Filters: Analysis and Design*, McGraw-Hill Book Co., Inc., New York (1979).

APPENDIX A

for n even. For n odd, there is one real pole, and the transfer function is of the form

$$H_{BP}(z^{-1}) = \frac{A_0(1 - z^{-2})}{1 - \alpha(1 - B_1)z^{-1} - B_1z^{-2}}$$

$$+ (1 - z^{-2}) \sum_{k=1}^{(n-1)/2} \left(\frac{A'_{11k}z^{-1} + A'_{01k}}{1 + B'_{11k}z^{-1} + B'_{21k}z^{-2}} \right.$$

$$\left. - \frac{A'_{12k}z^{-1} - A'_{02k}}{1 - B'_{12k}z^{-1} + B'_{22k}z^{-2}} \right).$$

A-9. BAND-STOP TRANSFORMATION

The procedure for the band-stop transformation is the same as for the band-pass transformation. In general, the transformation is

$$z^{-1} \rightarrow \frac{z^{-2} - \frac{2\alpha}{1+k}z^{-1} + \frac{1-k}{1+k}}{\frac{1-k}{1+k}z^{-2} - \frac{2\alpha}{1+k}z^{-1} + 1},$$

where

$$\alpha = \frac{\cos\pi(F_1 + F_2)T}{\cos\pi(F_2 - F_1)T},$$

$$k = \tan [\pi(F_2 - F_1)T] \tan (\pi\beta T).$$

By letting $k = 1$, then $\beta = F_s/2 - (F_2 - F_1)$, and the transformation is simplified.

$$z^{-1} \rightarrow z^{-1} \left(\frac{z^{-1} - \alpha}{1 - \alpha z^{-1}} \right).$$

APPENDIX A

The resulting band-stop transfer function has the form

$$H_{BS}(z^{-1}) = (1 - 2\alpha z^{-1} + z^{-2}) \sum_{k=1}^{n/2} \left(\frac{A'_{11k} z^{-1} - A'_{01k}}{1 - B'_{11k} z^{-1} + B'_{21k} z^{-2}} \right. \\ \left. - \frac{A'_{12k} z^{-1} + A'_{02k}}{1 + B'_{12k} z^{-1} + B'_{22k} z^{-2}} \right)$$

for n even. For n odd, the transfer function has the form

$$H_{BS}(z^{-1}) = \frac{(1 - 2\alpha z^{-1} + z^{-2})A_0}{1 - \alpha(1 + B_1)z^{-1} + B_1 z^{-2}} \\ + (1 - 2\alpha z^{-1} + z^{-2}) \sum_{k=1}^{(n-1)/2} \left(\frac{A'_{11k} z^{-1} - A'_{01k}}{1 - B'_{11k} z^{-1} + B'_{21k} z^{-2}} \right. \\ \left. - \frac{A'_{12k} z^{-1} + A'_{02k}}{1 + B'_{12k} z^{-1} + B'_{22k} z^{-2}} \right).$$

A-10. FILTER IMPLEMENTATION

From the z domain general transfer equation

$$H(z^{-1}) = (1 + z^{-1}) \sum_{i=1}^r \frac{A_{0i} + A_{1i} z^{-1}}{1 + B_{1i} z^{-1} + B_{2i} z^{-2}},$$

a time-domain difference equation is obtained. The difference equation is then used to process data samples.

$$Y_{out}(kT) = X(kT) + X[(k - 1)T],$$

APPENDIX A

where

$$X(kT) = \sum_{i=1}^r X_i(kT),$$

$$\begin{aligned} X_i(kT) &= A_{0i} Y_{in}(kT) + A_{1i} Y_{in}[(k-1)T] \\ &\quad - B_{1i} X_i[(k-1)T] - B_{2i} X_i[(k-2)T]. \end{aligned}$$

APPENDIX B.--FORTRAN IV COMPUTER PROGRAM FOR RECURSIVE DIGITAL FILTERS

CONTENTS

	<u>Page</u>
B-1. MAIN PROGRAM	51
B-2. SUBROUTINE RDATA	52
B-3. SUBROUTINE BUTTER	53
B-4. SUBROUTINE FACTOR	53
B-5. SUBROUTINE CHEB	54
B-6. SUBROUTINE CAUER	55
B-7. SUBROUTINE HIGHP	57
B-8. SUBROUTINE FPLOT	57
B-9. SUBROUTINE BANDP	59
B-10. SUBROUTINE BANDS	60
B-11. SUBROUTINE FILIMP	62
B-12. SUBROUTINE PLTDAT	63

The main program and all of the subroutines for recursive digital filters are included in this appendix. The only exceptions are standard library functions and the plotting routine. If a machine other than an IBM is used, it may be necessary to modify the format and data statements.

B-1. MAIN PROGRAM

```

REAL*8 A0(20), A1(20), B1(20), B2(20), PI,A2
REAL*8 AMP
COMPLEX*16 P(20), Z(20)
COMMON FC, FS, EPS1, EPS2, PI, ITYPE, N, ITRANS
COMMON/VDATA/VIN(1000), VOUT(1000),NTIMES
COMMON/TRANS/ F1,F2
COMMON/FILT/ WC, W1,W2,AMP,A0,A1,A2, B1, B2, P, Z
C ITYPE- FILTER- BUTTERWORTH(1), CHEBYSHEV(2), ELLIPTIC(3)
C N - ORDER OF FILTER, 20 MAX.
C ITRANS- NONE(0), HP(1), BP(2), BS(3)
C FC- CUTOFF FREQUENCY
C FS- SAMPLE FREQUENCY
C EPS1 - MINIMUM AMPLITUDE IN PASSBAND
C EPS2 - TRANSITION COEFFICIENT FOR CAUER, 0.95 MAX.
C F1 - LOWER CUTOFF FREQUENCY, PASS AND STOP FILTERS ONLY
C F2 - UPPER CUTOFF FREQUENCY, PASS AND STOP FILTERS ONLY
C NTIMES - NUMBER OF DATA POINTS
C VIN- INPUT DATA
C VOUT- OUTPUT DATA
P3=4.0D0*DATAN(1.0D0)
CALL RDATA
WC=2*DTAN(PI*FC/FS)
W1=PI*F1/FS
W2=PI*F2/FS
GO TO (10,20,30),ITYPE
10 CALL BUTTER
GO TO 40
20 CALL CHEB
GO TO 40
30 CALL CAUER
40 CALL FACTOR
CALL FPLOT
JJ=ITRANS+1
GO TO (100,50,60,70),JJ
50 CALL HIGHP
GO TO 80

```

APPENDIX B

B-1. MAIN PROGRAM (Cont'd)

```
60    CALL BANDP
      GO TO 80
70    CALL BANDS
80    CALL FPLOT
100   IF(NTIMES.EQ.0)GO TO 110
      CALL FILIMP
      CALL PLTDAT
110   CONTINUE
      STOP
      END
```

B-2. SUBROUTINE RDATA

```
SUBROUTINE RDATA
REAL#8 PI
COMMON FC, FS, EPS1, EPS2, PI, ITYPE, N, ITRANS
COMMON/VDATA/VIN(1000), VOUT(1000), NTIMES
COMMON/TRANS/ F1,F2
READ(5,10) ITYPE, N, ITRANS, FC,FS,EPS1, EPS2
10  FORMAT(3I5, 4E10.3)
F2=0
F1=F2
IF (ITRANS.EQ.0) GO TO 100
READ(5,20) F1,F2
20  FORMAT(2E10.3)
READ(5,25)NTIMES
25  FORMAT(I5)
IF(NTIMES.EQ.0)GO TO 50
JJ=(NTIMES+7)/8
DO 100 K=1,JJ
L=(K-1)*8+1
M=K*8
100 READ(5,30)(VIN(J),J=L,M)
30  FORMAT(8E10.3)
50  CONTINUE
      WRITE(6,45)
45  FORMAT(//,,5X5HITYPE,7X1HN,6X6HITRANS,9X2HFC,13X2HFS,12X4HEPS1,
111X4HEPS2,12X2HF1,13X2HF2)
      WRITE(6,40) ITYPE, N, ITRANS, FC, FS, EPS1, EPS2, F1, F2
40  FORMAT(/,3(5X15), 6(5XE10.3),///)
      RETURN
      END
```

APPENDIX B

B-3. SUBROUTINE BUTTER

```
SUBROUTINE BUTTER
REAL*8 A0(20), A1(20), B1(20), B2(20), P1,PHI
COMPLEX*16 P(20), Z(20)
COMMON FC, FS, EPS1, EPS2, P1, ITYPE, N, ITRANS
COMMON/FILT/WC, W1,W2,AMP,A0,A1,A2, B1, B2, P, Z
REAL*8 AMP,A2
A=N
K=N-2*INT(N/2.)
DO 20 I=1,N
B=J
IF(K.EQ.0) GO TO 5
PHI=P1*(A+1)/2.0+B-1.0/A
GO TO 10
5 PHI=P1*(1.0/(2.0*A)+(B-1.0)/A+0.5)
P11=WC*DCMPLX(DCOS(PHI),DSIN(PHI))
CONTINUE
AMP=WC**N
RETURN
END
```

B-4. SUBROUTINE FACTOR

```
SUBROUTINE FACTOR
REAL*8 AMP
REAL*8 A0(2 ), A1(20), B1(20), B2(20), P1, RR,R1,D, AK, BK,A2
COMPLEX*16 P(20), Z(20), R(20), X
COMMON FC, FS, EPS1, EPS2, P1, ITYPE, N, ITRANS
COMMON/FILT/WC, W1,W2,AMP,A0,A1,A2, B1, B2, P, Z
A2=1.0D0
K= INT((N+1)/2.)
DO 50 I=1,K
X=DCMPLX(1.0D0, 0.0D0)
DO 20 J=1,N
IF (J.EQ.I) GO TO 20
X=X*(P(I)-P(J))
20 CONTINUE
R(I)= AMP/X
IF(N.EQ.1)GO TO 30
X=DCMPLX(1.0D0,0.0D0)
IF(ITYPE.NE.3)GO TO 50
KK=2*(N/2)
DO 30 J=1,KK
X=X*(P(I)-Z(J))
```

APPENDIX B

B-4. SUBROUTINE FACTOR (Cont'd)

```

30  CONTINUE
    R(I)=R(I)*X
    AMP=AMP*(KK+1-N)
50  CONTINUE
    IF(ITYPE.NE.3)AMP=0.000
    DO 100 I=1,K
    RR=DREAL(R(I))
    RI=DIMAG(R(I))
    AK=-DREAL(P(I))+0.500
    BK= DIMAG(P(I))+0.500
    D=(1.000+AK)**2+BK**2
    A0(I)=(RR*(1.000+AK)-RI*BK)/D
    A1(I)=-(RR*(1.000-AK)+RI*BK)/D
    B1(I)= -2.000*(1.000-AK**2-BK**2)/D
    B2(I)=(1.000-AK)**2+BK**2)/D
100  CONTINUE
    I=2*K-N
    IF(I.EQ.0)GO TO 110
    A0(K)=A0(K)/2
    B1(K)=B1(K)/2
    A1(K)=0.
    B2(K)=0.
110  RETURN
    END

```

B-5. SUBROUTINE CHEB

```

SUBROUTINE CHEB
COMMON/FILT/WC, N1, N2, AMP, A0, A1, A2, B1, B2, P, Z
COMMON FC, FS, EPS1, EPS2, PI, ITYPE, N, ITRANS
COMPLEX*16 P(20), Z(20)
REAL*8 A0(20), A1(20), B1(20), B2(20)
REAL*8 CHV, SHV, V, J, AMP, PI, A2
EPS1=SQRT(1.0/EPS1**2-1.0)
A2=1.000
A=N
V=DLOG(1.000/EPS1+DSQRT(1.000*1.000/EPS1**2))/A
CHV=(DEXP(V)+DEXP(-V))/2.000
SHV=(DEXP(V)-DEXP(-V))/2.000
JJ=(N+1)/2
DO 50 I=1,N
U=P1*(2.000*(I+N)-1.000)/(2.000*A)
P(I)=WC*DCMPLX(DSIN(U)*CHV, -DCOS(U)*CHV)

```

APPENDIX B

B-5. SUBROUTINE CHEB (Cont'd)

```

50  CONTINUE
K=N-2*JJ
AMP=1.000
IF(K.EQ.0)GO TO 90
JJ=JJ-1
IF(N.EQ.1)GO TO 80
DO 75 I=1,JJ
AMP=AMP*(DREAL(P(I))**2+DIMAG(P(I)))**2
75  CONTINUE
80  AMP=AMP*DREAL(P(JJ+1))
GO TO 125
90  DO 100 I=1,JJ
AMP=AMP*(DREAL(P(I))**2+DIMAG(P(I)))**2
100 CONTINUE
AMP=AMP/DSQRT(1.000*EPS1**2)
125 RETURN
END

```

B-6. SUBROUTINE CAUER

```

SUBROUTINE CAUER
COMMON/FILT/WC, W1, W2, AMP, A0, A1, A2, B1, B2, P, Z
COMMON FC, FS, EPS1, EPS2, PI, ITYPE, N, ITRANS
REAL*8 AMP, Q, A, S0, W, O, V
REAL*8 A0(20), A1(20), B1(20), B2(20), PI, A2
COMPLEX*16 Z(20), P(20)
S0=0.000
W=0.000
Q=(1.000-DBLE(EPS2)**2)**(0.25)
C=0.500*((1.000-Q)/(1.000+Q))
AMP=1.000
Q=Q+2.0*Q**5+15.0*Q**9+150.0*Q**13
A=DBLE(EPS1)
A=DLOG((1.000/A+1.000)/(1.000/A-1.000))/(2.0*N)
O=0.000
DO 10 I=1,100
M=I-1
S0=S0+(-1.000)**M*Q**((M+1)*DCOSH(2.0*I*A))
IF(W.EQ.S0)GO TO 20
10 W=S0
20 DO 30 I=1,100
O=O+(-1.000)**I*Q**((I+1)*DCOSH(2.0*I*A))
IF(O.EQ.W)GO TO 40
30 W=0

```

APPENDIX B

B-6. SUBROUTINE CAUER (Cont'd)

```

4)      S0=DABS(2.+Q**10.25)*S0/(1.000+2.0*D)
C=N
W=DSQRT((1.+DC*EPS2*S0**2)*(1.000+S0**2/EPS2))
IF(N.EQ.1)GO TO 100
JJ=N/2
U=(2*JJ+1-N)/2.0
DO 100 I=1,JJ
D=0.000
V=0
DO 50 J=1,100,2
M=J-1
D=(-1.0)**M*Q**((M+J)*DSIN((M+J)*PI*(FLOAT(I)-U)/C)+0
M=J+1
D=(-1.0)**J*Q**((M+J)*DSIN((M+J)*PI*(FLOAT(I)-U)/C)+0
IF(D.EQ.0)GO TO 60
5)      V=0
6)      A=0.000
V=A
DO 70 J=1,100,2
M=J
V=V+(-1.0)**M*Q**((M+M)*DCOS(2.0*M*PI*(FLOAT(I)-U)/C)
M=J+1
V=V+(-1.0)**M*Q**((M+M)*DCOS(2.0*M*PI*(FLOAT(I)-U)/C)
IF(V.EQ.A)GO TO 80
7)      A=V
80      D=2.0*Q**((0.25)*D/(1.000+2.0*V)
V=(1.000-EPS2*D*D)*(1.000-D*D/EPS2)
V=DSQRT(V)
Z()=DCMPLX(0.000,1.000/D)*WC
Z(N+1-1)=DCONJG(Z())
A=1.000+(S0*D)**2
P()=DCMPLX(-S0*V/A,D*W/A)*WC
P(N+1-1)=DCONJG(P())
AMP=AMP*D*D*((S0*V)**2+(D*W)**2)/A**2
100    CONTINUE
K=N-2*(N/2)
IF(K.EQ.0)GO TO 150
P((N+1)/2)=DCMPLX(-S0*WC,0.000)
AMP=AMP*S0*WC
GO TO 200
150    AMP=AMP*EPS1
200    CONTINUE
RETURN
END

```

APPENDIX B

B-7. SUBROUTINE HIGHP

```

SUBROUTINE HIGHP
REAL*8 A0(2), A1(20), B1(20), B2(20), PI
REAL*8 AMP,A2
REAL*8 AP,A0D,A11,B11,B22,D
COMMON FC, FS, EPS1, EPS2, P1, ITYPE, N, ITRANS
COMMON/FILT/WC, W1,W2,AMP,A0,A1,A2, B1, B2, P, Z
W3=P1*FC/FS
A2=-1.000
AP=-COS(W1+W3)/COS(W1-W3)
DO 100 K=1,N
D=1.000-B1(K)*AP+B2(K)*AP*AP
A0D=(A0(K)-A1(K)*AP)*(1.000-AP)/D
A11=(A0(K)*AP-A1(K))*(1.000-AP)/D
B11=(AP*AP-B1(K)*(1.000*AP*AP)+2.000*B2(K)*AP)/D
B22=(AP*AP-AP*B1(K)+B2(K))/D
A0(K)=A0D
A1(K)=A11
B1(K)=B11
B2(K)=B22
100 CONTINUE
RETURN
END

```

B-8. SUBROUTINE FPLOT

```

SUBROUTINE FPLOT
COMMON FC, FS, EPS1, EPS2, P1, ITYPE, N, ITRANS
COMMON/FILT/WC, W1,W2,AMP,A0,A1,A2, B1, B2, P, Z
REAL*8 AMP
REAL*8 A0(20), A1(20), B1(20), B2(20), PI,A2
DIMENSION F(251),HEJNT(251),PHI(251)
DIMENSION XLAB(2),YLAB1(2),YLAB2(2),HEAD(10),SUBH(2)
DATA YLAB2(1),YLAB2(2)/8PHASE /D,BMEG/
DATA YLAB1(1),YLAB1(2)/8HAMPLSTUD,BHE /
DATA XLAB(1),XLAB(2)/8HFREQUENC,BHY /F/FS)/
WRITE(6,800)
JJ=(N+1)/2
DO 805 I=1,JJ
805 WRITE(6,801)A0(I),A1(I),B1(I),B2(I)
WRITE(6,802)AMP,A2
800 FORMAT(16X2HA'',32X2HA1,32X2HB1,32X2HB2,///)
801 FORMAT(1P4(1XQ32.25))

```

APPENDIX B

B-8. SUBROUTINE FPLOT (Cont'd)

```

802  FORMAT(4X6HAMP = ,1P032.25,10X5HA2 = ,0PF4.0)
JJ=(N+1)/2
READ(5,10) (HEAD(I),I=1,10)
10  FORMAT(10A8)
DO 100 I=1,250
A=I-1
F(I)=A/249.0
W=2.0*PI*F(I)
C=COS(W)
S=SIN(W)
PH1(I)=0.
HEJNT(I)=0.
DO 50 K=1,JJ
CN=A0(K)+A1(K)*C
DN=-A1(K)*S
CD=1.0+B1(K)*C+32(K)*(2.0*C-C-1.0)
DD=B1(K)*S+2.0*B2(K)*C*S
DD=-DD
EE=CD*CD+DD*DD
FF=(CN*(CD+DN*DD))/EE
EE=(DN*(CD-DD*CN))/EE
HEJNT(I)=HEJNT(I)+FF
PH1(I)=PH1(I)+EE
50  CONTINUE
FF=(1.0D0*A2*C)*HEJNT(I)+S*PH1(I)*A2      +AMP
EE=(1.0D0*A2*C)*PH1(I)-S*HEJNT(I)*A2
HEJNT(I)=SQRT(EE*EE+FF*FF)
PH3(I)=ATAN2(EE,FF)
PH3(I)=PH3(I)*180.0/PI
100 CONTINUE
CALL DRAWID(1,4,4,20,0,250,2.,0.,XLAB,
1           YLAB1,HEAD,SUBH,F,HEJNT)
F(I)=0.0
CALL DRAWID(1,4,4,20,0,250,2.0,0.,XLAB,
1YLAB2,HEAD,SUBH,F,PH1)
RETURN
END

```

```

SUBROUTINE BANDP
COMMON/FILT/WC, W1, W2, AMP, A0, A1, A2, B1, B2, P, Z
COMMON FC, FS, EPS1, EPS2, PI, ITYPE, N, ITRANS
REAL*B A0(20), A1(20), B1(20), B2(20), PI, A2, AMP
COMPLEX*36 Z(20), P(20), PP, Q, R, S, T, U, V
REAL*B C, D, AL, E, F
A2=0.000
AL=DCOS(DBLE(W1+W2))/DCOS(DBLE(W2-W1))
JJ=N/2
IF(N.EQ.1)GO TO 50
DO 50 I=1, JJ
AMP=AMP+A0(I)
C=B2(I)-B1(I)*Z*2/4.000
IF(C.LE.0.000)GO TO 40
PP=DCMPLX(-B1(I)/2.000, DSQRT(8C))
Q=(A0(I))*PP+PP*(A1(I)+A1(I)*PP+DCMPLX(A1(I), 0.000))
1/DCMPLX(0.000, 2.000*DIMAG(PP))
V=DCMPLX(1.000, 0.000)
S=AL*(V+PP)/2.000
R=S+CDSQRT(S*S-PP)
S=S-CDSQRT(S*S-PP)
T=0*(AL*R-V)/(R-S)
U=Q*(AL*S-V)/(S-R)
C=DREAL(R)
D=DIMAG(R)
E=DREAL(T)
F=DIMAG(T)
F=-2.0*(C*E+D*F)
D=C*C*D*D
C=-2.0*C
E=2.0*E
AMP=AMP+F/D
A0(I)=-F/D
A1(I)=E-F*C/D
B1(I)=C
B2(I)=D
K=(N+1)/2+1
C=DREAL(S)
D=DIMAG(S)
E=DREAL(U)
F=DIMAG(U)
F=-2.0*(C*E+D*F)
D=C*C*D*D
C=-2.0*C
E=2.0*E
AMP=AMP+F/D
A0(K)=-F/D
A1(K)=E-F*C/D
B1(K)=C
B2(K)=D
GO TO 50

```

APPENDIX B

B-9. SUBROUTINE BANDP (Cont'd)

```

40    C=DSQRT(1-C)
      D=-B1(1)/2.000*C
      C=-B1(1)/2.000-C
      E=(AO(1)*C+(A1(1)*A1(1))*C+A1(1)*C)/8(C-D)
      F=(AO(1)*D+D*(A1(1)*A1(1))*D+A1(1)*D)/8(D-C)
      AMP=AMP-E/C-F/D
      AO(1)=E/C
      A1(1)=-AL*E/C
      B1(1)=-AL*(1.0D)+C
      B2(1)=C
      K=(N+1)/2+1
      AO(K)=F/D
      A1(K)=-AL*F/D
      B1(K)=-AL*(1.0D)+D
      B2(K)=D
50    CONTINUE
      K=N-2*JJ
      IF(K.EQ.0)GO TO 100
      AMP=AMP+AO(N)/B1(N)
      D=B1(N)
      C=AO(N)*(1.0D-1.0/D)
      K=(N+1)/2
      AO(K)=C
      A1(K)=-AL*C
      B1(K)=AL*(D-1.0D)
      B2(K)=-D
100   N=2*N
      RETURN
      END

```

B-10. SUBROUTINE BANDS

SUBROUTINE BANDS

COMMON/FILT/W1,W2,AMP,AO,A1,A2,B1,B2,P,Z
 COMMON FC,FS,EPS1,EPS2,PI,ITYPE,N,ITRANS
 REAL*8 AO(20),A1(20),B1(20),B2(20),PI,A2,AMP
 COMPLEX*16 Z(20),P(20),PP,Q,R,S,T,U,V
 REAL*8 C,D,AL,E,F,X,C1,C2
 A2=0.000
 AL=DCOS(DBLE(W1+W2))/DCOS(DBLE(W2-W1))
 JJ=N/2
 X=DTAN(DBLE(W2-W1))*DTAN(PI)*FC/FS
 C1=2.0*AL/(X+1.0D)
 C2=(1.0D-X)/(X+1.0D)

APPENDIX B

B-10. SUBROUTINE BANDS (Cont'd)

```

IF(N.EQ.3)GO TO 50
DO 50 I=1,JJ
V=DCMPLX(1.0D0,0.0D0)
AMP=AMP+AD(I)
C=B2(I)-B1(I)**2/4.0D0
IF(C.LE.0.0D0)GO TO 40
PP=DCMPLX(-B1(I)/2.0D0,DSQRT(K))
Q=(AD(I)*PP+PP*(AD(I)+A1(I)))*PP+DCMPLX(A1(I),0.0D0)
I/DCMPLX(0.0D0,2.0D0*DIMAG(PP))
Q=Q+C2/(V-PP+C2)
AMP=AMP+2.0*DREAL(Q)
S=(1+(V-PP)/(V-C2*PP)*2.0)
T=(C2*V-PP)/(V-C2*PP)
R=S+CDSQRT(S+S-T)
S=S-CDSQRT(S+S-T)
T=Q*(R-R-C1/C2*S+V/C2)/(R-S)
U=Q*(S+S-C1/C2*S+V/C2)/(S-R)
C=DREAL(R)
D=DIMAG(R)
E=DREAL(T)
F=DIMAG(T)
F=-2.0*(C+E+D+F)
D=C*C+D*D
C=-2.0*C
E=2.0*E
AMP=AMP+F/D
AD(I)=-F/D
A1(I)=E-F*C/D
B1(I)=C
B2(I)=D
K=(N+1)/2+1
C=DREAL(S)
D=DIMAG(S)
E=DREAL(U)
F=DIMAG(U)
F=-2.0*(C+E+D+F)
D=C*C+D*D
C=-2.0*C
E=2.0*E
AMP=AMP+F/D
AD(K)=-F/D
A1(K)=E-F*C/D
B1(K)=C
B2(K)=D
GO TO 50

```

APPENDIX B

B-10. SUBROUTINE BANDS (Cont'd)

```

40   C=DSQRT(6-C)
      D=-B1(I)/2.0D0+C
      C=-B1(I)/2.0D0-C
      E=(A0(I)*C*(C+(A0(I)+A1(I))*C+A1(I)))/(C-D)
      F=(A0(I)*D*D+(A0(I)+A1(I))*D+A1(I))/(D-C)
      AMP=AMP+E/(C2-C)+F/(C2-D)
      A0(I)=E*(C2/(1.0D0+C2)-1.0/(C2-C))
      A1(I)=-E*(1*(C+C2)/(1.0D0+C2)**2)
      B1(I)=C1*(C-1.0D0)/(1.0D0+C2)
      B2(I)=(C2-C)/(1.0D0+C2)
      K=I+N
      B2(K)=(C2-D)/(1.0D0+C2)
      B1(K)=C1*(D-1.0D0)/(1.0D0+C2)
      A0(K)=F*(C2/(1.0D0+C2)-1.0/(C2-D))
      A1(K)=-F*(1*(D+C2)/(1.0D0+C2)**2)
50 CONTINUE
      K=N-2*JJ
      IF(K.EQ.0) GO TO 100
      K=(N+1)/2
      AMP=AMP+A0(K)
      D=B1(K)
      C=A0(K)/(1.0D0+B1(K)*C2)*(1.0D0-D)
      B2(K)=(C2+D)/(1.0D0+C2*D)
      B1(K)=-C1*(D+1.0D0)/(1.0D0+C2*D)
      AMP=AMP+C/B2(K)
      A0(K)=C*C2-(/B2(K))
      A1(K)=-C1*(C-B1(K))/B2(K)
100 N=2*NN
      RETURN
      END

```

B-11. SUBROUTINE FILIMP

```

SUBROUTINE FILIMP
COMMON FC, FS, EPS1, EPS2, P3, ITYPE, N, ITRANS
REAL*B AMP
REAL*B A0(20), A1(20), B1(20), B2(20), PI,A2
COMMON/VDATA/ VIN(1000),VOUT(1000),NTIMES
COMMON/FILT/WC, W1,W2,AMP,A0,A1,A2, B1, B2, P, Z
REAL*B YK(2,20), YSUM1, YSUM2, YTEM
YSUM1=0.0
YSUM2=0.0
JJ=(N+1)/2
DO 10 K=1, JJ

```

APPENDIX B

B-11. SUBROUTINE FILIMP (Cont'd)

```

YK(2,K)=AO(K)*VIN(1)
YK(1,K)=AO(K)*VIN(2)+(A1(K)-B1(K)*AO(K))*VIN(1)
YSUM2=YSUM2+YK(2,K)
YSUM1=YSUM1+YK(1,K)
10 CONTINUE
VOUT(1)=YSUM2
VOUT(2)=YSUM1
DO 50 J=3,NTIMES
YSUM2=YSUM1
YSUM1=0.0
DO 40 K=3, JJ
YTEM=AO(K)*VIN(J)+A1(K)*VIN(J-1)
1-B1(K)*YK(1,K)-B2(K)*YK(2,K)
YK(2,K)=YK(1,K)
YK(1,K)=YTEM
YSUM1=YSUM1+YTEM
40 CONTINUE
VOUT(J)=YSUM1+YSUM2*A2      +AMP*VIN(J)
50 CONTINUE
RETURN
END

```

B-12. SUBROUTINE PLTDAT

```

SUBROUTINE PLTDAT
COMMON/VDATA/VIN(1000),VOUT(1000),NTIMES
COMMON FC,FS
DIMENSION HEAD(10),XLAB(2),YLAB(2),T(1000)
DATA XLAB(1),XLAB(2)/$HTIME (SE,BHCOND$) /
DATA SUB1,SUB2/$HVIN   ,$HVOUT  /
DATA YLAB(1),YLAB(2)/$HAMPLITUD,BHE    /
DO 20 I=1,NTIMES
20 T(I)=(I-1)/FS
READ(5,10)(HEAD(I)),I=1,10
10 FORMAT(10A8)
CALL DRAW1D(1,4,4,20,2,NTIMES,2.,0.,XLAB,YLAB,HEAD,SUB1,T,VIN)
READ(5,10)(HEAD(I)),I=1,10
CALL DRAW1D(1,4,4,20,2,NTIMES,2.,0.,XLAB,YLAB,HEAD,SUB2,T,VOUT)
RETURN
END

```

DISTRIBUTION

ADMINISTRATOR
DEFENSE TECHNICAL INFORMATION CENTER
ATTN DTIC-DOA (12 COPIES)
CAMERON STATION, BUILDING 5
ALEXANDRIA, VA 22314

COMMANDER
US ARMY RSCH & STD GP (EUR)
ATTN CHIEF, PHYSICS & MATH BRANCH
FPO NEW YORK 09510

COMMANDER
US ARMY MISSILE & MUNITIONS
CENTER & SCHOOL
ATTN ATSK-CTD-F
REDSTONE ARSENAL, AL 35809

DIRECTOR
US ARMY MATERIEL SYSTEMS ANALYSIS
ACTIVITY
ATTN DRXSY-MP
ATTN DRXSY-PO
ABERDEEN PROVING GROUND, MD 21005

DIRECTOR
US ARMY BALLISTIC RESEARCH
LABORATORY
ATTN DRDAR-TSB-S (STINFO)
ATTN DRXBR-AM, W. VANANTWERP
ATTN DRSTE-EL
ATTN DRDAR-BLE
ABERDEEN PROVING GROUND, MD 21005

US ARMY ELECTRONICS TECHNOLOGY
AND DEVICES LABORATORY
ATTN DELET-DD
FT MONMOUTH, NJ 07703

HO, USAF/SAMI
WASHINGTON, DC 20330

TELEDYNE BROWN ENGINEERING
CUMMINGS RESEARCH PARK
ATTN DR. MELVIN L. PRICE, MS-44
HUNTSVILLE, AL 35807

DIRECTOR
ARMED FORCES RADIobiology
RESEARCH INSTITUTE
DEFENSE NUCLEAR AGENCY
NATIONAL NAVAL MEDICAL CENTER
ATTN RESEARCH PROGRAM
COORDINATING OFFICER
BETHESDA, MD 20014

ASSISTANT TO THE SECRETARY OF
DEFENSE
ATOMIC ENERGY
ATTN EXECUTIVE ASSISTANT
WASHINGTON, DC 20301

DIRECTOR
DEFENSE ADVANCED RSCH PROJ AGENCY
ARCHITECTURE BUILDING
1400 WILSON BLVD
ATTN TIO
ARLINGTON, VA 22201

FEDERAL EMERGENCY MANAGEMENT AGENCY
ATTN JAMES W. KEEF
Mitigation & Research
WASHINGTON, DC 20472

DIRECTOR
DEFENSE COMMUNICATIONS AGENCY
ATTN CODE C312
ATTN CODE C313
WASHINGTON, DC 20305

DEFENSE COMMUNICATIONS ENGINEERING
CENTER
1860 WIEHLE AVENUE
ATTN CODE R720, C. STANSBERRY
ATTN CODE R123, RSCH LIB
ATTN CODE R400
RESTON, VA 22090

DIRECTOR
DEFENSE INTELLIGENCE AGENCY
ATTN RDS-3A
ATTN RDS-3A4, POMPONIO PLAZA
WASHINGTON, DC 20301

DIRECTOR
DEFENSE NUCLEAR AGENCY
ATTN DNST, E. E. CONRAD, DEP DIR
SCI & TECH
ATTN RAEV, ELECTRONICS VULNERABILITY
DIV
ATTN TITL, TECHNICAL LIBRARY DIV
ATTN RAEE, EMP EFFECTS DIV
WASHINGTON, DC 20305

COMMANDER
FIELD COMMAND
DEFENSE NUCLEAR AGENCY
ATTN FCPR
ATTN FCSPM, J. SMITH
ATTN FCLMC
KIRKLAND AFB, NM 87115

DIRECTOR
INTERSERVICE NUCLEAR WEAPONS SCHOOL
ATTN TTV
KIRTLAND AFB, NM 87115

JOINT CHIEFS OF STAFF
ATTN J-3
WASHINGTON, DC 20301

DIRECTOR
JOINT STRATEGIC TARGET PLANNING
STAFF, JCS
ATTN JSAS
ATTN JPST
ATTN NF1-STINFO LIBRARY
OFFUTT AFB
OMAHA, NE 68133

CHIEF
LIVERMORE DIVISION, FIELD COMMAND
DNA
LAWRENCE LIVERMORE LABORATORY

PO BOX 808
ATTN FPLRL
LIVERMORE, CA 94550

NATIONAL COMMUNICATIONS SYSTEM
OFFICE OF THE MANAGER
DEPARTMENT OF DEFENSE
ATTN NCS-TS, CHARLES D. BOISON
WASHINGTON, DC 20305

DIRECTOR
NATIONAL SECURITY AGENCY
DEPARTMENT OF DEFENSE
ATTN R-52, O. VAN GUNTEN
ATTN S232, D. VINCENT
FT GEORGE G. MEAD, MD 20755

UNDER SECRETARY OF DEF FOR RSCH &
ENGRG
DEPARTMENT OF DEFENSE
ATTN G. BARSE
ATTN S4SS (OS)
WASHINGTON, DC 20301

COMMANDER
BMD SYSTEM COMMAND
DEPARTMENT OF THE ARMY
PO BOX 1500
ATTN BMDS-C-AOLIB
HUNTSVILLE, AL 35807

COMMANDER
ERADCOM TECHNICAL SUPPORT ACTIVITY
DEPARTMENT OF THE ARMY
ATTN DRDCO-COM-ME, G. GAULE
ATTN TECHNICAL LIBRARY DIV
ATTN DELLS-K, A. COHEN
ATTN DELET-IR, E. HUNTER
FT. MONMOUTH, NJ 07703

COMMANDER
US ARMY ARMOR CENTER
ATTN TECHNICAL LIBRARY
FT KNOX, KY 40121

COMMANDER
US ARMY COMM-ELEC ENGRG INSTAL.
AGENCY
ATTN CCC-PROS-S
ATTN CCC-CED-SES
FT HUACHICA, AZ 85613

COMMANDER
US ARMY COMMUNICATIONS COMMAND
ATTN CY-OPS-PD
ATTN CY-OPS-OS
FT HUACHICA, AZ 85613

COMMANDER
US ARMY COMMUNICATIONS COMMAND
COMBAT DEVELOPMENT DIVISION
ATTN ATSI-D-MD
FT HUACHICA, AZ 85613

CHIEF
US ARMY COMMUNICATIONS SYS AGENCY
ATTN CCM-RD-T, CCM-AD-SV
FT MONMOUTH, NJ 07703

PROJECT OFFICER
US ARMY COMMUNICATIONS RES
A DEV COMMAND
ATTN DRDM-ATM
ATTN DRDM-TDS-JSL
FT MONMOUTH, NJ 07703

DIVISION ENGINEER
US ARMY ENGINEER DIV, HUNTSVILLE
PO BOX 1600, WEST STATION
ATTN HNED-DR
ATTN A. T. BOLT
HUNTSVILLE, AL 35807

DISTRIBUTION (Cont'd)

US ARMY INTEL THREAT ANALYSIS DETACHMENT ROOM 2201, BLDG A ARLINGTON HALL STATION ATTN RM 2200, BLDG A ARLINGTON, VA 22212	DIRECTOR NAVAL RESEARCH LABORATORY ATTN CODE 4104, EMANAL L. BRANCATO ATTN CODE 2627, DORIS R. FOLEN ATTN CODE 6623, RICHARD L. STATLER ATTN CODE 6624 WASHINGTON, DC 20375	DIRECTOR AIR UNIVERSITY LIBRARY DEPARTMENT OF THE AIR FORCE ATTN AUL-LSE-70-250 MAXWELL AFB, AL 36112
COMMANDER US ARMY INTELLIGENCE & SEC CMD ARLINGTON HALL STATION 4000 ARLINGTON BLVD ATTN TECHNICAL LIBRARY ATTN TECH INFO FAC ARLINGTON, VA 22212	COMMANDER NAVAL SHIP ENGINEERING CENTER DEPARTMENT OF THE NAVY ATTN CODE 6174D2, EDWARD F. DUFFY WASHINGTON, DC 20362	HEADQUARTERS ELECTRONIC SYSTEMS DIVISION/YSEA DEPARTMENT OF THE AIR FORCE ATTN YSEA HANSOM AFB, MA 01731
COMMANDER US ARMY MISSILE COMMAND ATTN DRCM-PE-EA, WALLACE O. WAGNER ATTN DRCM-PE-EG, WILLIAM R. JOHNSON ATTN DRMI-TBD ATTN DRMI-EAA REDSTONE ARSENAL, AL 35809	COMMANDER NAVAL SURFACE WEAPONS CENTER ATTN CODE WAS1RH, RM 130-108 ATTN CODE F32, EDWIN R. RATHBURN WHITE OAK, SILVER SPRING, MD 20910	COMMANDER FOREIGN TECHNOLOGY DIVISION, AFSC ATTN NTOP, R. L. HALLARD WRIGHT-PATTERSON AFB, OH 45433
COMMANDER US ARMY TEST AND EVALUATION COMMAND ATTN DSTE-FA ABERDEEN PROVING GROUND, MD 21005	COMMANDER NAVAL WEAPONS CENTER ATTN CODE 533, TECH LIB CHINA LAKE, CA 93555	COMMANDER OGDEN AIR MMEDDE DEPARTMENT OF THE AIR FORCE ATTN (X-AIR) MMETH, P. W. BERTHEL ATTN MMEDDE, LEO KIDMAN ATTN MAJ R. BLACKBURN HILL AFB, UT 84406
COMMANDER US ARMY TRAINING AND DOCTRINE COMMAND ATTN ATOTI-OP-SW PT MONROE, VA 23651	COMMANDING OFFICER NAVAL WEAPONS EVALUATION FACILITY KIRTLAND AIR FORCE BASE ATTN CODE AT-6 ALBUQUERQUE, NM 87117	COMMANDER ROME AIR DEVELOPMENT CENTER, AFSC ATTN TSLO GRIFFISS AFB, NY 13441
COMMANDER WHITE SANDS MISSILE RANGE DEPARTMENT OF THE ARMY ATTN STEWS-TE-AN, J. OKIMA WHITE SANDS MISSILE RANGE, NM 88002	OFFICE OF NAVAL RESEARCH ATTN CODE 427 ARLINGTON, VA 22217	COMMANDER SACRAMENTO AIR LOGISTICS CENTER DEPARTMENT OF THE AIR FORCE ATTN MMRS, H. A. PELMASTRO ATTN MMRA, J. W. DEMES ATTN MMREM, F. R. SPEAR MCLELLAN AFB, CA 95652
OFFICER-IN-CHARGE CIVIL ENGINEERING LABORATORY NAVAL CONSTRUCTION BATTALION CENTER ATTN CODE LOBA (LIBRARY) ATTN CODE LOBA PORT HUENEME, CA 93041	DIRECTOR STRATEGIC SYSTEMS PROJECT OFFICE NAVY DEPARTMENT ATTN NSP-2701, JOHN W. PITSENBERGER ATTN NSP-2342, RICHARD L. COLEMAN ATTN NSP-43, TECH LIB ATTN NSP-27334 ATTN NSP-240, LT. COL. WASHINGTON, DC 20376	SAMSON IN AIR FORCE SYSTEMS COMMAND PO BOX 92960 WORLDWAY INSTAL CENTER (INTELLIGENCE) ATTN IND LOS ANGELES, CA 90009
COMMANDER NAVAL AIR SYSTEMS COMMAND ATTN AIR-350F WASHINGTON, DC 20340	COMMANDER AERONAUTICAL SYSTEMS DIVISION, AFSC ATTN ASI-YH-EX ATTN ENFTV WRIGHT-PATTERSON AFB, OH 45433	SAMSON MN AIR FORCE SYSTEMS COMMAND (MINUTEMAN) ATTN MNH, MAJ M. BARAN ATTN MNH, CAPT R. L. LAWRENCE NOFTN AFB, CA 94049
COMMANDER NAVAL ELECTRONIC SYSTEMS COMMAND ATTN NME 117-215 WASHINGTON, DC 20340	AIR FORCE TECHNICAL APPLICATIONS CENTER ATTN TEC, M. SCHNEIDER PATRICK AFB, FL 33599	SAMSON VA AIR FORCE SYSTEMS COMMAND PO BOX 92960 WORLDWAY INSTAL CENTER ATTN YAH LOS ANGELES, CA 90009
COMMANDER NAVAL SYSTEMS CENTER ATTN COMCOTS, C. FLETCHER ATTN CODE 7240, S. W. LICHMAN SAN DIEGO, CA 92152	AF WEAPON LABORATORY, AFSC ATTN NTN ATTN NT ATTN EL, CARL E. BAUM ATTN ELX ATTN SEL ATTN CA ATTN ELA, J. P. CARTER ATTN ELI ATTN ELS, W. PAGE ATTN NXS Kirtland AFB, NM 87117	STRATEGIC AIR COMMAND XEIS ATTN NHI-STINNEY LIBRARY ATTN DEL ATTN GARNET E. MATTHE ATTN XEIS, MAJ RICHAN G. STEPHAN REFLT AFB, ND 58511
COMMANDING OFFICER NAVAL ORDNANCE STATION ATTN STANDARDIZATION DIV INDIAN HEAD, MD 20640		DEPARTMENT OF ENERGY ARMED FORCES OPERATIONS CENTER PO BOX 54000 ATTN TD-H LIBRARY ATTN OPERATIONAL SAFETY DIV ALBUQUERQUE, NM 87188
SUPERINTENDENT (CODE 1424) NAVAL POSTGRADUATE SCHOOL ATTN CODE 1424 MONTEREY, CA 93940		

DISTRIBUTION (Cont'd)

UNIVERSITY OF CALIFORNIA LAWRENCE LIVERMORE LABORATORY PO BOX 808 ATTN TECH INFO DEPT ATTN L-06, T. DONTCH ATTN L-545, D. MEKKER ATTN L-156, E. MILLER ATTN L-10, H. KRUGER ATTN H. S. CARAYAN LIVERMORE, CA 94550	HDM CORPORATION PO BOX 9274 ALBUQUERQUE INTERNATIONAL ATTN LIB ALBUQUERQUE, NM 87119	CONTROL DATA CORPORATION PO BOX 0 ATTN JACK MEEHAN MINNEAPOLIS, MN 55440
LOS ALAMOS SCIENTIFIC LABORATORY PO BOX 1663 ATTN BRUCE W. NOEL ATTN CLARENCE BENTON LOS ALAMOS, NM 87545	BENDIX CORPORATION RESEARCH LABORATORIES DIVISION BENDIX CENTER ATTN MAX FRANK SOUTHFIELD, MI 48075	CUTLER-HAMMER, INC. AII. DIVISION COMAC ROAD ATTN EDWARD KARPEN DEER PARK, NY 11729
SANDIA LABORATORIES PO BOX 5800 ATTN C. N. VITTIONE ATTN R. L. PARKER ATTN ELMER F. HARTMAN ALBUQUERQUE, NM 87115	BENDIX CORPORATION NAVIGATION AND CONTROL GROUP ATTN DEPT 6401 TETERBORO, NJ 07608	DIKEWOOD CORPORATION 1613 UNIVERSITY BLVD., NE ATTN TECH LIB ATTN L. WAYNE DAVIS ALBUQUERQUE, NM 87102
CENTRAL INTELLIGENCE AGENCY ATTN RVST, RM 5G48, HQ, BLDG (COST/NED/NWU) WASHINGTON, DC 20505	BOEING COMPANY PO BOX 3707 ATTN HOWARD W. WICKLEIN ATTN D. E. ISBELL ATTN DAVID KEMBLE ATTN R. C. HANRAHAN ATTN KENT TECH LIB SEATTLE, WA 98124	DIKENKOD CORPORATION 2716 OCEAN & PARK BLVD., SUITE 6000 ATTN K. LEE SANTA MONICA, CA 90405
ADMINISTRATOR DEFENSE ELECTRIC POWER ADMIN DEPARTMENT OF THE INTERIOR INTERIOR SOUTH BLDG, 312 ATTN L. GUNNELL WASHINGTON, DC 20240	BROWN ENGINEERING COMPANY, INC. CUMMINGS RESEARCH PARK ATTN FRED LEONARD HUNTSVILLE, AL 35807	E-SYSTEMS, INC. GREENVILLE DIVISION PO BOX 1056 ATTN JOELTA MOORE GREENVILLE, TX 75401
DEPARTMENT OF TRANSPORTATION FEDERAL AVIATION ADMINISTRATION HEADQUARTERS SEC DIV, ASF-300 5000 INDEPENDENCE AVENUE, SW ATTN SEC DIV ASF-300 WASHINGTON, DC 20591	BURROUGHS CORPORATION FEDERAL AND SPECIAL SYSTEMS GROUP CENTRAL AVE AND ROUTE 252 PO BOX 517 ATTN ANGELO J. MAURIELLO PAOLI, PA 19301	EFFECTS TECHNOLOGY, INC. 5383 HOLLISTER AVENUE ATTN S. CLOW SANTA BARBARA, CA 93111
AEROSPACE CORPORATION PO BOX 92957 ATTN C. R. PEARLSTON ATTN IRVING M. GARFUNKEL ATTN JULIAN REINHEIMER ATTN LIBRARY ATTN CHARLES GREENHOW LOS ANGELES, CA 90009	CALSPAN CORPORATION PO BOX 400 ATTN TECH LIBRARY ROCHESTER, NY 14626	EXXON NUCLEAR COMPANY, INC. RESEARCH AND TECHNICAL CENTER 2955 GEORGE WASHINGTON WAY ATTN DR. A. W. TRUVELLE RICHLAND, WA 99352
ANARIAN ASSOCIATES 250 NORTH NASH STREET ATTN LIBRARY EL SEGUNDO, CA 90245	CHARLES STARK DRAPER LABORATORY, INC. 555 TECHNOLOGY SQUARE ATTN KENNETH FERTIG ATTN TIC MS 74 CAMBRIDGE, MA 02139	FAIRCHILD CAMERA AND INSTRUMENT CORP. 464 ELLIS STREET ATTN DAVID K. MYERS MOUNTAIN VIEW, CA 94039
AVCO RESEARCH & SYSTEMS GROUP 301 LAUREL STREET WILMINGTON, MA 01887	CINCINNATI ELECTRONICS CORPORATION 2630 GLENDALE - MILFORD ROAD ATTN LOIS HAMMOND CINCINNATI, OH 45241	FORD AEROSPACE & COMMUNICATIONS CORP. 3939 FAIRLANE WAY ATTN TECHNICAL LIBRARY FAIRLANE, MI 48136
BAPTIST MEMORIAL INSTITUTE 505 KING AVENUE ATTN ROBERT H. BLASER ATTN EUGENE R. LEACH COLUMBUS, OH 43201	COMPUTER SCIENCES CORPORATION 6565 ARLINGTON BLVD. ATTN RAMONA BRIGGS FALLS CHURCH, VA 22046	FORD AEROSPACE & COMMUNICATIONS CORP. OPERATIONS 1400 S. JAMBOREE ROAD ATTN KEN CO. ATTINGER ATTN E. G. PONCELET, JR. NEWPORT BEACH, CA 92660
BEI CORPORATION 141 JONES BRANCH DRIVE ATTN CORPORATE LIBRARY MCLEAN, VA 22102	COMPUTER SCIENCES CORPORATION 1400 SAN MARINO BLVD., SE ATTN RICHARD H. LICKHAGE ATTN ALVIN J. HIFF ALBUQUERQUE, NM 87109	FRANKLIN INSTITUTE 20TH & BROAD AND PARKWAY ATTN RAMON B. THOMAS PHILADELPHIA, PA 19103

DISTRIBUTION (Cont'd)

GENERAL DYNAMICS CORP ELECTRONICS DIVISION PO BOX 81125 ATTN RSCH LIB SAN DIEGO, CA 92138	GTE SYLVANIA, INC 189 B. STREET ATTN CHARLES H. RAMSBOTTOM ATTN DAVID D. FLOOD ATTN EMIL P. MOTCHOK ATTN H & V GROUP, MARIO A. NUREFORA ATTN J. WALDRON NEEDHAM HEIGHTS, MA 02194	JAYCOR 11011 TORREYANA ROAD PO BOX 85154 ATTN ERIC P. WENAAS ATTN RALPH H. STAHL SAN DIEGO, CA 92138
GENERAL DYNAMICS CORPORATION INTER-DIVISION RESEARCH LIBRARY KEARNY MESA PO BOX 80847 ATTN RESEARCH LIBRARY SAN DIEGO, CA 98123	HARRIS CORPORATION HARRIS SEMICONDUCTOR DIVISION PO BOX 883 ATTN V PRES & MGR PRGMS DIV MELBOURNE, FL 32901	JAYCOR 205 S. WHITTING STREET, SUITE 500 ATTN LIB ALEXANDRIA, VA 22304
GENERAL ELECTRIC CO.-TEMPO CENTER FOR ADVANCED STUDIES 816 STATE STREET (PO DRAWER QO) ATTN DASIA ATTN ROYDEN R. RUTHERFORD ATTN WILLIAM MCNAMEERA SANTA BARBARA, CA 93102	HAZELTINE CORPORATION PULASKI ROAD ATTN TECH INFO CTR, M. WAITE GREENLAWN, NY 11740	KAMAN SCIENCES CORPORATION 1500 GARDEN OF THE GODS ROAD ATTN ALBERT P. BRIDGES ATTN W. FOSTER RICH ATTN WALTER E. WARE ATTN FRANK H. SHELTON ATTN JERRY I. LURELL ATTN PHIL. TRACY COLORADO SPRINGS, CO 80907
GENERAL ELECTRIC COMPANY AEROSPACE ELECTRONICS SYSTEMS FRENCH ROAD ATTN CHARLES M. HEWISON UTICA, NY 13503	HONEYWELL INCORPORATED AVIONICS DIVISION 2600 RIDGEWAY PARKWAY ATTN S&R LIB ATTN RONALD R. JOHNSON MINNEAPOLIS, MN 55413	LITTON SYSTEMS, INC DATA SYSTEMS DIVISION 8000 WOODLEY AVENUE ATTN EMC GP ATTN M848-61 VAN NUYS, CA 91409
GENERAL ELECTRIC COMPANY PO BOX 5000 ATTN TECH LIB BINGHAMTON, NY 13902	HONEYWELL INCORPORATED AVIONICS DIVISION 13350 U.S. HIGHWAY 19 NORTH ATTN MS 725-5, STACEY H. GRAFF ATTN W. E. STEWART ST PETERSBURG, FL 33733	LITTON SYSTEMS, INC AMERCOM DIVISION 5115 CALVERT ROAD ATTN J. SKAGGS COLLEGE PARK, MD 20740
GENERAL ELECTRIC CO.-TEMPO ALEXANDRIA OFFICE HUNTINGTON BUILDING, SUITE 300 2560 HUNTINGTON AVENUE ATTN DASIA ALEXANDRIA, VA 22303	HUGHES AIRCRAFT COMPANY CENTINELA AND TEALE ATTN JOHN B. SINGLETON ATTN UTDC 6/E110 ATTN KENNETH R. WALKER CULVER CITY, CA 90230	LOCKHEED MISSILES AND SPACE CO., INC PO BOX 504 ATTN L. ROSSI ATTN SAMUEL L. TAIMITY ATTN H. E. THAYN ATTN GEORGE F. HEATH ATTN BENJAMIN T. KIMURA SUNNYVALE, CA 94086
GENERAL RESEARCH CORPORATION SANTA BARBARA DIVISION PO BOX 6770 ATTN TECH INFO OFFICE SANTA BARBARA, CA 93111	IIT RESEARCH INSTITUTE ELECTROMAG COMPATABILITY ANAL CTR NORTH SEVERN ATTN ACOAT ANNAPOLIS, MD 21402	LOCKHEED MISSILE AND SPACE COMPANY, INC 3751 HANOVER STREET ATTN TECH INFO CTR D/C/EL PALO ALTO, CA 94304
GEORGIA INSTITUTE OF TECHNOLOGY GEORGIA TECH RESEARCH INSTITUTE ATTN R. TURRY ATLANTA, GA 30332	IIT RESEARCH INSTITUTE 10 WEST 35TH STREET ATTN IRVING N. MINDEL ATTN JACK E. BRIDGES CHICAGO, IL 60616	M.I.T. LINCOLN LABORATORY PO BOX 73 ATTN LEONA DRAUGHIN LEXINGTON, MA 02173
GEORGIA INSTITUTE OF TECHNOLOGY OFFICE OF CONTRACT ADMINISTRATION ATTN RSCH SECURITY COORDINATOR FOR RICH DENNY ATLANTA, GA 30332	INSTITUTE FOR DEFENSE ANALYSIS 400 ARMY-NAVY DRIVE ATTN TECH INFO SERVICES ARLINGTOM, VA 22202	MARTIN MARIETTA CORPORATION ORLANDO DIVISION PO BOX 5637 ATTN MONA C. GRIFFITH ORLANDO, FL 32805
GRUMMAN AEROSPACE CORPORATION SOUTH OYSTER BAY ROAD ATTN L-01 35 BETHLEHEM, NY 11714	INT'L TEL & TELEGRAPH CORPORATION 500 WASHINGTON AVENUE ATTN TECHNICAL LIBRARY ATTN ALEXANDER T. RICHARDSON NUTLEY, NJ 07110	MCDONNELL DOUGLAS CORPORATION PO BOX 516 ATTN TOM ENGER ST. LOUIS, MO 63166
GTE SYLVANIA, INC ELECTRONICS SYSTEMS GRD-EASTERN DIV 77 A STREET ATTN CHARLES A. THORNHILL, LIBRARIAN ATTN LEONARD L. BLAISDELL NEEDHAM, MA 02194	IRT CORPORATION PO BOX 8707 ATTN P. R. WILLIAMS ATTN DENNIS SWIFT SAN DIEGO, CA 92148	MCDONNELL DOUGLAS CORPORATION 9401 BELINA AVENUE ATTN STANLEY SCHNEIDER ATTN TECH LIBRARY SERVICES HUNTINGTON BEACH, CA 92647

DISTRIBUTION (Cont'd)

MISSION RESEARCH CORPORATION
PO DRAWER 710
ATTN ENFO GROUP
ATTN WILLIAM C. BART
ATTN V. LUNDBERG
SANTA BARBARA, CA 93140

MISSION RESEARCH CORPORATION
PO BOX 9816
ATTN WERNER STARK
ATTN RAY STRAYER, JR.
CULPEPER SPRINGS, NC 28623

MISSION RESEARCH CORPORATION
FM SYSTEM APPLICATIONS DIVISION
1400 SAN MATEO BLVD., SUITE A
ATTN DAVID E. MCKEEWETHER
ALBUQUERQUE, NM 87108

MISSION RESEARCH CORPORATION—
SAN DIEGO
PO BOX 1209
ATTN V. A. J. VAN LINT
LA JOLLA, CA 92038

MITRE CORPORATION
PO BOX 208
ATTN M. P. FITZGERALD
BEDFORD, MA 01730

NORDEN SYSTEMS, INC.
HELEN STREET
ATTN TECHNICAL LIBRARY
NEWARK, DE 19856

NORTHROP RESEARCH TECHNOLOGY CENTER
ONE RESEARCH PARK
ATTN LIBRARY
PALOS VERDES PENN, CA 90274

NORTHERN CORPORATION
ELECTRONIC DIVISION
2401 WEST 120TH STREET
ATTN LEW SMITH
ATTN RAD EFFECTS GRP
HAWTHORNE, NJ 07030

PHYSICS INTERNATIONAL COMPANY
2200 MERCER STREET
ATTN DUC CON
SAN LEANDRO, CA 94577

SAC ASSOCIATES
PO BOX 4605
ATTN K. CLAY ROBERTS
ATTN CHARLES M.
ATTN RICHARD E. SCHAEFER
ATTN DON CONN
ATTN M. S. COOPER
ATTN J. M. COONAN
ATTN J. B. MARSHALL
MADISON, WI 53701

SANE CORPORATION
101 MAIN STREET
ATTN LINDA
ATTN G. MULFREY
SANTA MONICA, CA 90406

RAYTHEON COMPANY
HAWTHORNE, NJ
ATTN ANTHONY H. JONES
ROUTE 1, MA 01730

RAYTHEON COMPANY
528 BOSTON POST ROAD
ATTN HAROLD L. FLEISCHER
SUDSBURY, MA 01776

RCA CORPORATION
GOVERNMENT SYSTEMS DIVISION
ASTRO ELECTRONICS
PO BOX 800, LOCUST CORNER
EAST WINDSOR TOWNSHIP
ATTN GEORGE J. BROOKER
PRINCETON, NJ 08540

RCA CORPORATION
DAVID SARNOFF RESEARCH CENTER
PO BOX 432
ATTN SECURITY DEPT. L. MINICH
PRINCETON, NJ 08540

RCA CORPORATION
CAMDEN COMPLEX
FRONT & COOPER STREETS
ATTN OLIVE WHITEHEAD
ATTN R. W. ROSTROM
CAMDEN, NJ 08105

ROCKWELL INTERNATIONAL CORP
PO BOX 3105
ATTN M. J. RUDIE
ATTN J. L. MONROE
ATTN V. J. MICHEL
ATTN D-243-068, 031-C-011
ANAHEIM, CA 92803

ROCKWELL INTERNATIONAL CORP
SPACE DIVISION
12214 SOUTH LAKEWOOD BOULEVARD
ATTN R. E. WHITE
DOWNNEY, CA 90241

ROCKWELL INTERNATIONAL CORPORATION
815 LAKHAM STREET
ATTN RSL, DIV TIC (BAM)
EL SEGUNDO, CA 90245

ROCKWELL INTERNATIONAL CORPORATION
PO BOX 300
ATTN E. A. SHAW
CLEARFIELD, UT 84015

RANGER ASSOCIATES, INC.
10 CANAL STREET
ATTN J. L. COOPER, JR., W. C. TAYLOR, JR., F. F.
NASHUA, NH 03063

SCIENCE APPLICATIONS, INC.
PO BOX 2227
ATTN DEPARTMENT M, TECH DEPT
BERKELEY, CA 94704

SCIENCE APPLICATIONS, INC.
PO BOX 1251
ATTN R. PARKINSON
LA JOLLA, CA 92038

SCIENCE APPLICATIONS, INC.
HUNTSVILLE DIVISION
2700 W. CLINTON AVENUE
SUITE 200
ATTN NOEL F. ROSEN
HUNTSVILLE, AL 35805

SCIENCE APPLICATIONS, INC.
8400 WESTHORN DRIVE
ATTN WILLIAM L. THAGARD
MCLEAN, VA 22101

SINGER COMPANY
1150 MERRIFIELD AVENUE
ATTN TECH INFO DEPT
CLIPPLE FIELD, NJ 07044

SPERRY RAND CORPORATION
SPERRY RADAR SYSTEMS
SPERRY MICRO WAVE ELECTRONICS
PO BOX 4606
ATTN MARKETING DEPT
CLEARWATER, FL 33565

SPERRY RAND CORPORATION
SLICKY DIVISION
MANUS AVENUE
ATTN TECH DEPT
GREAT NECK, NY 11020

SPERRY RAND CORPORATION
SPERRY FLIGHT SYSTEMS
PO BOX 22111
ATTN J. ANDREW SCHAFF
PRENTICE, MI 49660

SPIRE CORPORATION
PO BOX 11
ATTN JOHN R. KELLY
ATTN ROBERT M. LITTLE
BEDFORD, MA 01730

SRI INTERNATIONAL
333 RAVENSWOOD AVENUE
ATTN ARTHUR LEE WHITTON
MENLO PARK, CA 94034

SYSTEMS, SCIENTIFIC AND SOFTWARE, INC.
PO BOX 1001
ATTN ANDREW S. WILSON
LA JOLLA, CA 92038

Texas Instruments, Inc.
PO BOX 6000
ATTN TECH DEPT
ATTN DONALD J. MANTZ
DALLAS, TX 75229

TRW DEFENSE & SPACE SYSTEMS
100 STATE PARK
ATTN G. E. ADAMS
ATTN R. E. UPHAM
ATTN J. E. MANGELSEN
ATTN B. M. HOLLOWAY
ATTN W. J. KARBAR
KODIAK, AK 99610

TEXAS TECH UNIVERSITY
P.O. BOX 14474 N. 4TH COLLEGE STATION
ATTN TRAVIS L. SIMPSON
LUBBOCK, TX 79404

TRW, INC., INTEGRATED SYSTEMS
HAMILTON STANDARD DIVISION
KIRKLAND INTERNATIONAL AIRPORT
ATTN GENE FLETCHER
WINDSOR, CONNECTICUT 06095

DISTRIBUTION (Cont'd)

WESTINGHOUSE ELECTRIC CORPORATION
ADVANCED ENERGY SYSTEMS DIV
PO BOX 1064
ATTN TECH LIB
PITTSBURGH, PA 15236

US ARMY ELECTRONICS RESEARCH
& DEVELOPMENT COMMAND
ATTN TECHNICAL DIRECTOR, DRREL-C
ATTN LEGAL OFFICE, 97000
ATTN R. HARMON, DRREL-MA

HARRY DIAMOND LABORATORY
ATTN INC TL-TSG DIVISION DIRECTORS
ATTN RECORD COPY, 81200
ATTN HDL LIBRARY, 81100 (5 COPIES)
ATTN HDL LIBRARY, 81100 (WOODBRIDGE)
ATTN TECHNICAL REPORTS BRANCH, 81300
ATTN CHAIRMAN, EDITORIAL COMMITTEE
ATTN CHIEF, 21000
ATTN CHIEF, 21100
ATTN CHIEF, 21200
ATTN CHIEF, 21300 (5 COPIES)
ATTN CHIEF, 21400
ATTN CHIEF, 21500
ATTN CHIEF, 22000
ATTN CHIEF, 22100
ATTN CHIEF, 22300
ATTN CHIEF, 22600
ATTN CHIEF, 22900
ATTN CHIEF, 23240
ATTN WIMENITZ, E., 20240
ATTN BUTTLIN, G., 22900
ATTN CHASE, E., 21100
ATTN VALLIN, J., 22100
ATTN FEMENIAS, R., 22100
ATTN LEEDER, K., 22100
ATTN REYER, P., 21300
ATTN LANHAM, C., 00210
ATTN PEPPERONE, S., 36000
ATTN CHIEF, 13000
ATTN BENSLER, M., 15300
ATTN LEWIS, D., 15300
ATTN SHREEVE, J., 15300
ATTN HIEHL, R., 15300
ATTN SEROL, E., 36200 (5 COPIES)
ATTN BANN, K., 15300
ATTN EMMERMAN, P., 15400
ATTN FEMENIAS, R., 22100
ATTN GRAY, R., 21300 (5 COPIES)